CONTENTS

CHAPTER 1. THE ELECTROSTATIC FIELD IN VACUUM

1–1 Vector fields ........................................ 1
1–2 The electric field .................................... 7
1–3 Coulomb's law ........................................ 8
1–4 The electrostatic potential ......................... 10
1–5 The potential in terms of charge distribution .... 11
1–6 Field singularities .................................. 13
1–7 Clusters of point charges ......................... 13
1–8 Dipole interactions ................................ 19
1–9 Surface singularities .............................. 20
1–10 Volume distributions of dipole moment ........ 23

CHAPTER 2. BOUNDARY CONDITIONS AND RELATION OF MICROSCOPIC TO MACROSCOPIC FIELDS

2–1 The displacement vector .......................... 28
2–2 Boundary conditions ............................... 31
2–3 The electric field in a material medium ....... 33
2–4 Polarizability ........................................ 38

CHAPTER 3. GENERAL METHODS FOR THE SOLUTION OF POTENTIAL PROBLEMS

3–1 Uniqueness theorem ............................... 42
3–2 Green's reciprocation theorem ................... 43
3–3 Solution by Green's function .................... 44
3–4 Solution by inversion ............................. 47
3–5 Solution by electrical images .................... 49
3–6 Solution of Laplace's equation by the separation of variables 53

CHAPTER 4. TWO-DIMENSIONAL POTENTIAL PROBLEMS

4–1 Conjugate complex functions .................... 61
4–2 Capacity and field strength ..................... 63
4–3 The potential of a uniform field ............... 64
4–4 The potential of a line charge ................. 64
4–5 Complex transformations ....................... 66
4–6 General Schwarz transformation ............... 67
4–7 Single-angle transformations .................... 70
4–8 Multiple-angle transformations ................ 71
4–9 Direct solution of Laplace's equation by the method of harmonics 73
4–10 Illustration: Line charge and dielectric cylinder 74
4–11 Line charge in an angle between two conductors 77
CHAPTER 5. THREE-DIMENSIONAL POTENTIAL PROBLEMS

5-1 The solution of Laplace's equation in spherical coordinates . 81
5-2 The potential of a point charge . 82
5-3 The potential of a dielectric sphere and a point charge . 83
5-4 The potential of a dielectric sphere in a uniform field . 84
5-5 The potential of an arbitrary axially-symmetric spherical potential distribution . 86
5-6 The potential of a charged ring . 87
5-7 Problems not having axial symmetry . 88
5-8 The solution of Laplace's equation in cylindrical coordinates . 88
5-9 Application of cylindrical solutions to potential problems . 91

CHAPTER 6. ENERGY RELATIONS AND FORCES IN THE ELECTROSTATIC FIELD

6-1 Field energy in free space . 95
6-2 Energy density within a dielectric . 98
6-3 Thermodynamic interpretation of $U$ . 100
6-4 Thomson's theorem . 101
6-5 Maxwell stress tensor . 103
6-6 Volume forces in the electrostatic field in the presence of dielectrics . 107
6-7 The behavior of dielectric liquids in an electrostatic field . 111

CHAPTER 7. STEADY CURRENTS AND THEIR INTERACTION

7-1 Ohm's law . 118
7-2 Electromotive force . 119
7-3 The solution of stationary current problems . 120
7-4 Time of relaxation in a homogeneous medium . 122
7-5 The magnetic interaction of steady line currents . 123
7-6 The magnetic induction field . 125
7-7 The magnetic scalar potential . 125
7-8 The magnetic vector potential . 127
7-9 Types of currents . 129
7-10 Polarization currents . 129
7-11 Magnetic moments . 130
7-12 Magnetization and magnetization currents . 134
7-13 Vacuum displacement current . 135

CHAPTER 8. MAGNETIC MATERIALS AND BOUNDARY VALUE PROBLEMS

8-1 Magnetic field intensity . 139
8-2 Magnetic sources . 140
8-3 Permeable media: magnetic susceptibility and boundary conditions . 144
8-4 Magnetic circuits . 145
### CONTENTS

8–5 Solution of boundary value problems by magnetic scalar potentials .................................................. 146
8–6 Uniqueness theorem for the vector potential ......................................................................................... 147
8–7 The use of the vector potential in the solution of problems ................................................................. 148
8–8 The vector potential in two dimensions .................................................................................................. 151
8–9 The vector potential in cylindrical coordinates ..................................................................................... 153

**CHAPTER 9. MAXWELL’S EQUATIONS** ..................................................................................................... 158
9–1 Faraday’s law of induction ..................................................................................................................... 158
9–2 Maxwell’s equations for stationary media ............................................................................................... 159
9–3 Faraday’s law for moving media ............................................................................................................ 160
9–4 Maxwell’s equations for moving media .................................................................................................. 163
9–5 Motion of a conductor in a magnetic field ............................................................................................... 165

**CHAPTER 10. ENERGY, FORCE, AND MOMENTUM RELATIONS IN THE ELECTROMAGNETIC FIELD** .......................................................... 170
10–1 Energy relations in quasi-stationary current systems ........................................................................... 170
10–2 Forces on current systems .................................................................................................................... 172
10–3 Inductance ............................................................................................................................................ 174
10–4 Magnetic volume force .......................................................................................................................... 177
10–5 General expressions for electromagnetic energy .................................................................................. 178
10–6 Momentum balance ............................................................................................................................... 181

**CHAPTER 11. THE WAVE EQUATION AND PLANE WAVES** ................................................................. 185
11–1 The wave equation ............................................................................................................................... 185
11–2 Plane waves .......................................................................................................................................... 187
11–3 Radiation pressure .................................................................................................................................. 191
11–4 Plane waves in a moving medium ......................................................................................................... 193
11–5 Reflection and refraction at a plane boundary ....................................................................................... 195
11–6 Waves in conducting media and metallic reflection ............................................................................ 200
11–7 Group velocity ....................................................................................................................................... 202

**CHAPTER 12. CONDUCTING FLUIDS IN A MAGNETIC FIELD (MAGNETOHYDRODYNAMICS)** ..................................................................... 205
12–1 “Frozen-in” lines of force ..................................................................................................................... 205
12–2 Magnetohydrodynamic waves .............................................................................................................. 207

**CHAPTER 13. WAVES IN THE PRESENCE OF METALLIC BOUNDARIES** ................................................ 212
13–1 The nature of metallic boundary conditions .......................................................................................... 212
13–2 Eigenfunctions and eigenvalues of the wave equation ......................................................................... 214
13–3 Cavities with rectangular boundaries .................................................................................................... 218
13–4 Cylindrical cavities .................................................................................................................................. 219
13–5 Circular cylindrical cavities .................................................................................................................... 222
13–6 Wave guides ............................................................................................................................................ 223
13–7 Scattering by a circular cylinder ............................................................................................................ 226
CONTENTS

13-8 Spherical waves ........................................... 229
13-9 Scattering by a sphere .................................. 233

CHAPTER 14. THE INHOMOGENEOUS WAVE EQUATION .......... 240
14-1 The wave equation for the potentials .................. 240
14-2 Solution by Fourier analysis ............................ 242
14-3 The radiation fields ..................................... 245
14-4 Radiated energy ......................................... 248
14-5 The Hertz potential ..................................... 254
14-6 Computation of radiation fields by the Hertz method . 255
14-7 Electric dipole radiation ................................ 257
14-8 Multipole radiation ...................................... 260
14-9 Derivation of multipole radiation from scalar superpotentials 264
14-10 Energy and angular momentum radiated by multipoles .. 267

CHAPTER 15. THE EXPERIMENTAL BASIS FOR THE THEORY OF
SPECIAL RELATIVITY ............................................. 272
15-1 Galilean relativity and electrodynamics ............... 272
15-2 The search for an absolute ether frame ............... 274
15-3 The Lorentz-Fitzgerald contraction hypothesis ........ 278
15-4 “Ether drag” ............................................. 279
15-5 Emission theories ....................................... 280
15-6 Summary ............................................... 283

CHAPTER 16. RELATIVISTIC KINEMATICS AND THE LORENTZ
TRANSFORMATION .................................................. 286
16-1 The velocity of light and simultaneity .................. 286
16-2 Kinematic relations in special relativity .............. 288
16-3 The Lorentz transformation .............................. 293
16-4 Geometric interpretations of the Lorentz transformation . 297
16-5 Transformation equations for velocity .................. 301

CHAPTER 17. COVARIANCE AND RELATIVISTIC MECHANICS .... 305
17-1 The Lorentz transformation of a four-vector .......... 305
17-2 Some tensor relations useful in special relativity .... 307
17-3 The conservation of momentum .......................... 311
17-4 Relation of energy to momentum and to mass .......... 313
17-5 The Minkowski force .................................... 316
17-6 The collision of two similar particles .................. 318
17-7 The use of four-vectors in calculating kinematic relations 
for collisions ................................................. 320

CHAPTER 18. COVARIANT FORMULATION OF ELECTRODYNAMICS .. 324
18-1 The four-vector potential ................................ 324
18-2 The electromagnetic field tensor ........................ 327
18-3 The Lorentz force in vacuum ............................. 331
CONTENTS

22-4 Scattering by a bound electron 407
22-5 Absorption of radiation by an oscillator 407
22-6 Equilibrium between an oscillator and a radiation field 409
22-7 Effect of a volume distribution of scatterers 411
22-8 Scattering from a volume distribution. Rayleigh scattering 414
22-9 The dispersion relation 416
22-10 A general theorem on scattering and absorption 419

CHAPTER 23. THE MOTION OF CHARGED PARTICLES IN ELECTROMAGNETIC FIELDS 425

23-1 World-line description 425
23-2 Hamiltonian formulation and the transition to three-dimensional formalism 427
23-3 Equations for the trajectories 430
23-4 Applications 433
23-5 The motion of a particle with magnetic moment in an electromagnetic field 437

CHAPTER 24. HAMILTONIAN FORMULATION OF MAXWELL’S EQUATIONS 446

24-1 Transition to a one-dimensional continuous system 446
24-2 Generalization to a three-dimensional continuum 448
24-3 The electromagnetic field 451
24-4 Periodic solutions in a box. Plane wave representation 454

APPENDIX I. UNITS AND DIMENSIONS IN ELECTROMAGNETIC THEORY 459

Tables: I-1. Conversion Factors 465
I-2. Fundamental Electromagnetic Relations Valid in vacuo as They Appear in the Various Systems of Units 466
I-3. Definition of Fields from Sources (mks system) 468
I-4. Useful Numerical Relations 469

APPENDIX II. USEFUL VECTOR RELATIONS 470

Table II-1. Vector Formulas 470

APPENDIX III. VECTOR RELATIONS IN CURVILINEAR COORDINATES 473

Table III-1. Coordinate Systems 475

BIBLIOGRAPHY 479

INDEX 485