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Statistical Analysis
of Extreme Values

from Insurance, Finance, Hydrology and Other Fields

Second edition

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