

SERIES ON UNIVERSITY MATHEMATICS – VOL. 1

Lectures on
**Differential
Geometry**

S. S. Chern

University of California, USA

W. H. Chen

Peking University, China

K. S. Lam

California State Polytechnic University, Pomona, USA

 **World Scientific**
Singapore • New Jersey • London • Hong Kong

Contents

1	Differentiable Manifolds	1
	§1-1 Definition of Differentiable Manifolds	1
	§1-2 Tangent Spaces	9
	§1-3 Submanifolds	18
	§1-4 Frobenius' Theorem	29
2	Multilinear Algebra	39
	§2-1 Tensor Products	39
	§2-2 Tensors	47
	§2-3 Exterior Algebra	52
3	Exterior Differential Calculus	65
	§3-1 Tensor Bundles and Vector Bundles	65
	§3-2 Exterior Differentiation	74
	§3-3 Integrals of Differential Forms	85
	§3-4 Stokes' Formula	92
4	Connections	101
	§4-1 Connections on Vector Bundles	101
	§4-2 Affine Connections	113
	§4-3 Connections on Frame Bundles	121
5	Riemannian Geometry	133
	§5-1 The Fundamental Theorem of Riemannian Geometry	133
	§5-2 Geodesic Normal Coordinates	143
	§5-3 Sectional Curvature	155
	§5-4 The Gauss-Bonnet Theorem	162
6	Lie Groups and Moving Frames	173
	§6-1 Lie Groups	173
	§6-2 Lie Transformation Groups	186
	§6-3 The Method of Moving Frames	198

§6-4 Theory of Surfaces	210
7 Complex Manifolds	221
§7-1 Complex Manifolds	221
§7-2 The Complex Structure on a Vector Space	227
§7-3 Almost Complex Manifolds	236
§7-4 Connections on Complex Vector Bundles	244
§7-5 Hermitian Manifolds and Kählerian Manifolds	256
8 Finsler Geometry	265
§8-1 Preliminaries	265
§8-2 Geometry on the Projectivised Tangent Bundle (<i>PTM</i>) and the Hilbert Form	267
§8-3 The Chern Connection	273
§8-3.1 Determination of the Connection	274
§8-3.2 The Cartan Tensor and Characterization of Riemannian Geometry	280
§8-3.3 Explicit Formulas for the Connection Forms in Natural Coordinates	283
§8-4 Structure Equations and the Flag Curvature	288
§8-4.1 The Curvature Tensor	289
§8-4.2 The Flag Curvature and the Ricci Curvature	293
§8-4.3 Special Finsler Spaces	295
§8-5 The First Variation of Arc Length and Geodesics	297
§8-6 The Second Variation of Arc Length and Jacobi Fields	306
§8-7 Completeness and the Hopf-Rinow Theorem	314
§8-8 The Theorems of Bonnet-Myers and Synge	325
A Historical Notes	331
§A-1 Classical Differential Geometry	331
§A-2 Riemannian Geometry	331
§A-3 Manifolds	332
§A-4 Global Geometry	332
B Differential Geometry and Theoretical Physics	335
§B-1 Dynamics and Moving Frames	336
§B-2 Theory of Surfaces, Solitons and the Sigma Model	338
§B-3 Gauge Field Theory	340
§B-4 Conclusion	341
References	343
Index	347