

CONTENTS

Part A. Ordinary Differential Equations.	1
Chapter 1 First-Order Differential Equations.	2
1.1 Basic Concepts and Ideas, 2	
1.2 Geometrical Meaning of $y' = f(x, y)$. Direction Fields, 10	
1.3 Separable Differential Equations, 14	
1.4 Modeling: Separable Equations, 19	
1.5 Exact Differential Equations. Integrating Factors, 25	
1.6 Linear Differential Equations. Bernoulli Equation, 33	
1.7 Modeling: Electric Circuits, 41	
1.8 Orthogonal Trajectories of Curves. <i>Optional</i> , 48	
1.9 Existence and Uniqueness of Solutions. Picard Iteration, 52	
<i>Chapter Review, 59</i>	
<i>Chapter Summary, 61</i>	
Chapter 2 Linear Differential Equations of Second and Higher Order	64
2.1 Homogeneous Linear Equations of Second Order, 64	
2.2 Second-Order Homogeneous Equations with Constant Coefficients, 72	
2.3 Case of Complex Roots. Complex Exponential Function, 76	
2.4 Differential Operators. <i>Optional</i> , 81	
2.5 Modeling: Free Oscillations (Mass–Spring System), 83	
2.6 Euler–Cauchy Equation, 93	
2.7 Existence and Uniqueness Theory. Wronskian, 97	
2.8 Nonhomogeneous Equations, 101	
2.9 Solution by Undetermined Coefficients, 104	
2.10 Solution by Variation of Parameters, 108	
2.11 Modeling: Forced Oscillations. Resonance, 111	
2.12 Modeling of Electric Circuits, 118	
2.13 Higher Order Linear Differential Equations, 124	
2.14 Higher Order Homogeneous Equations with Constant Coefficients, 132	
2.15 Higher Order Nonhomogeneous Equations, 138	
<i>Chapter Review, 142</i>	
<i>Chapter Summary, 143</i>	
Chapter 3 Systems of Differential Equations, Phase Plane, Qualitative Methods.	146
3.0 Introduction: Vectors, Matrices, Eigenvalues, 146	
3.1 Introductory Examples, 152	
3.2 Basic Concepts and Theory, 159	
3.3 Homogeneous Systems with Constant Coefficients. Phase Plane, Critical Points, 162	
3.4 Criteria for Critical Points. Stability, 170	
3.5 Qualitative Methods for Nonlinear Systems, 175	
3.6 Nonhomogeneous Linear Systems, 184	
<i>Chapter Review, 190</i>	
<i>Chapter Summary, 192</i>	

Chapter 4 Series Solutions of Differential Equations.	
Special Functions	194
4.1 Power Series Method, 194	
4.2 Theory of the Power Series Method, 198	
4.3 Legendre's Equation. Legendre Polynomials $P_n(x)$, 205	
4.4 Frobenius Method, 211	
4.5 Bessel's Equation. Bessel Functions $J_\nu(x)$, 218	
4.6 Bessel Functions of the Second Kind $Y_\nu(x)$, 228	
4.7 Sturm–Liouville Problems. Orthogonal Functions, 233	
4.8 Orthogonal Eigenfunction Expansions, 240	
<i>Chapter Review, 247</i>	
<i>Chapter Summary, 248</i>	
Chapter 5 Laplace Transforms	250
5.1 Laplace Transform. Inverse Transform. Linearity. Shifting, 251	
5.2 Transforms of Derivatives and Integrals. Differential Equations, 258	
5.3 Unit Step Function. Second Shifting Theorem. Dirac's Delta Function, 265	
5.4 Differentiation and Integration of Transforms, 275	
5.5 Convolution. Integral Equations, 279	
5.6 Partial Fractions. Differential Equations, 284	
5.7 Systems of Differential Equations, 291	
5.8 Laplace Transform: General Formulas, 296	
5.9 Table of Laplace Transforms, 297	
<i>Chapter Review, 299</i>	
<i>Chapter Summary, 302</i>	
Part B. Linear Algebra, Vector Calculus.	303
Chapter 6 Linear Algebra: Matrices, Vectors, Determinants.	
Linear Systems of Equations	304
6.1 Basic Concepts. Matrix Addition, Scalar Multiplication, 305	
6.2 Matrix Multiplication, 311	
6.3 Linear Systems of Equations. Gauss Elimination, 321	
6.4 Rank of a Matrix. Linear Independence. Vector Space, 331	
6.5 Solutions of Linear Systems: Existence, Uniqueness, General Form, 338	
6.6 Determinants. Cramer's Rule, 341	
6.7 Inverse of a Matrix. Gauss–Jordan Elimination, 350	
6.8 Vector Spaces, Inner Product Spaces, Linear Transformations. <i>Optional</i> , 358	
<i>Chapter Review, 365</i>	
<i>Chapter Summary, 367</i>	
Chapter 7 Linear Algebra: Matrix Eigenvalue Problems	370
7.1 Eigenvalues, Eigenvectors, 371	
7.2 Some Applications of Eigenvalue Problems, 376	
7.3 Symmetric, Skew-Symmetric, and Orthogonal Matrices, 381	
7.4 Complex Matrices: Hermitian, Skew-Hermitian, Unitary, 385	
7.5 Similarity of Matrices. Basis of Eigenvectors. Diagonalization, 392	
<i>Chapter Review, 398</i>	
<i>Chapter Summary, 399</i>	
Chapter 8 Vector Differential Calculus. Grad, Div, Curl.	400
8.1 Vector Algebra in 2-Space and 3-Space, 401	
8.2 Inner Product (Dot Product), 408	

8.3	Vector Product (Cross Product), 414
8.4	Vector and Scalar Functions and Fields. Derivatives, 423
8.5	Curves. Tangents. Arc Length, 428
8.6	Curves in Mechanics. Velocity and Acceleration, 435
8.7	Curvature and Torsion of a Curve. <i>Optional</i> , 440
8.8	Review from Calculus in Several Variables. <i>Optional</i> , 443
8.9	Gradient of a Scalar Field. Directional Derivative, 446
8.10	Divergence of a Vector Field, 453
8.11	Curl of a Vector Field, 457
	<i>Chapter Review</i> , 459
	<i>Chapter Summary</i> , 461

Chapter 9 Vector Integral Calculus. Integral Theorems 464

9.1	Line Integrals, 464
9.2	Line Integrals Independent of Path, 471
9.3	From Calculus: Double Integrals. <i>Optional</i> , 478
9.4	Green's Theorem in the Plane, 485
9.5	Surfaces for Surface Integrals, 491
9.6	Surface Integrals, 496
9.7	Triple Integrals. Divergence Theorem of Gauss, 505
9.8	Further Applications of the Divergence Theorem, 510
9.9	Stokes's Theorem, 515
	<i>Chapter Review</i> , 521
	<i>Chapter Summary</i> , 522

Part C. Fourier Analysis and Partial Differential Equations 525

Chapter 10 Fourier Series, Integrals, and Transforms 526

10.1	Periodic Functions. Trigonometric Series, 527
10.2	Fourier Series, 529
10.3	Functions of Any Period $p = 2L$, 537
10.4	Even and Odd Functions. Half-Range Expansions, 541
10.5	Complex Fourier Series. <i>Optional</i> , 547
10.6	Forced Oscillations, 550
10.7	Approximation by Trigonometric Polynomials, 553
10.8	Fourier Integrals, 557
10.9	Fourier Cosine and Sine Transforms, 564
10.10	Fourier Transform, 569
10.11	Tables of Transforms, 576
	<i>Chapter Review</i> , 579
	<i>Chapter Summary</i> , 580

Chapter 11 Partial Differential Equations 582

11.1	Basic Concepts, 583
11.2	Modeling: Vibrating String, Wave Equation, 585
11.3	Separation of Variables. Use of Fourier Series, 587
11.4	D'Alembert's Solution of the Wave Equation, 595
11.5	Heat Equation: Solution by Fourier Series, 600
11.6	Heat Equation: Solution by Fourier Integrals and Transforms, 610
11.7	Modeling: Membrane, Two-Dimensional Wave Equation, 616
11.8	Rectangular Membrane. Use of Double Fourier Series, 619
11.9	Laplacian in Polar Coordinates, 626
11.10	Circular Membrane. Use of Fourier-Bessel Series, 629

11.11	Laplace's Equation in Cylindrical and Spherical Coordinates. Potential,	636
11.12	Solution by Laplace Transforms,	643
	<i>Chapter Review,</i>	647
	<i>Chapter Summary,</i>	648
Part D.	Complex Analysis	651
	Chapter 12 Complex Numbers and Functions.	
	Conformal Mapping	652
12.1	Complex Numbers. Complex Plane,	652
12.2	Polar Form of Complex Numbers. Powers and Roots,	657
12.3	Derivative. Analytic Function,	663
12.4	Cauchy–Riemann Equations. Laplace's Equation,	669
12.5	Geometry of Analytic Functions: Conformal Mapping,	674
12.6	Exponential Function,	679
12.7	Trigonometric Functions, Hyperbolic Functions,	682
12.8	Logarithm. General Power,	687
12.9	Linear Fractional Transformations. <i>Optional,</i>	692
12.10	Riemann Surfaces. <i>Optional,</i>	699
	<i>Chapter Review,</i>	701
	<i>Chapter Summary,</i>	702
	Chapter 13 Complex Integration	704
13.1	Line Integral in the Complex Plane,	704
13.2	Cauchy's Integral Theorem,	713
13.3	Cauchy's Integral Formula,	721
13.4	Derivatives of Analytic Functions,	725
	<i>Chapter Review,</i>	730
	<i>Chapter Summary,</i>	731
	Chapter 14 Power Series, Taylor Series	732
14.1	Sequences, Series, Convergence Tests,	732
14.2	Power Series,	741
14.3	Functions Given by Power Series,	746
14.4	Taylor Series and Maclaurin Series,	751
14.5	Uniform Convergence. <i>Optional,</i>	759
	<i>Chapter Review,</i>	767
	<i>Chapter Summary,</i>	768
	Chapter 15 Laurent Series, Residue Integration	770
15.1	Laurent Series,	770
15.2	Singularities and Zeros. Infinity,	776
15.3	Residue Integration Method,	781
15.4	Evaluation of Real Integrals,	787
	<i>Chapter Review,</i>	794
	<i>Chapter Summary,</i>	796
	Chapter 16 Complex Analysis Applied to Potential Theory.	798
16.1	Electrostatic Fields,	799
16.2	Use of Conformal Mapping,	804
16.3	Heat Problems,	808
16.4	Fluid Flow,	812
16.5	Poisson's Integral Formula,	819
16.6	General Properties of Harmonic Functions,	822

Chapter Review, 826
Chapter Summary, 827

Part E. Numerical Methods 828

Software 829

Chapter 17 Numerical Methods in General. 830

- 17.1 Introduction: Floating Point. Round-off, Error Propagation, etc., 831
- 17.2 Solution of Equations by Iteration, 838
- 17.3 Interpolation, 848
- 17.4 Splines, 861
- 17.5 Numerical Integration and Differentiation, 869

Chapter Review, 882
Chapter Summary, 884

Chapter 18 Numerical Methods in Linear Algebra. 886

- 18.1 Linear Systems: Gauss Elimination, 886
- 18.2 Linear Systems: LU-Factorization, Matrix Inversion, 894
- 18.3 Linear Systems: Solution by Iteration, 900
- 18.4 Linear Systems: Ill-Conditioning, Norms, 906
- 18.5 Method of Least Squares, 914
- 18.6 Matrix Eigenvalue Problems: Introduction, 917
- 18.7 Inclusion of Matrix Eigenvalues, 920
- 18.8 Eigenvalues by Iteration (Power Method), 925
- 18.9 Tridiagonalization and QR-Factorization, 929

Chapter Review, 938
Chapter Summary, 940

Chapter 19 Numerical Methods for Differential Equations. 942

- 19.1 Methods for First-Order Differential Equations, 942
- 19.2 Multistep Methods, 952
- 19.3 Methods for Systems and Higher Order Equations, 956
- 19.4 Methods for Elliptic Partial Differential Equations, 962
- 19.5 Neumann and Mixed Problems. Irregular Boundary, 971
- 19.6 Methods for Parabolic Equations, 976
- 19.7 Methods for Hyperbolic Equations, 982

Chapter Review, 984
Chapter Summary, 987

Part F. Optimization, Graphs. 989

Chapter 20 Unconstrained Optimization, Linear Programming 990

- 20.1 Basic Concepts. Unconstrained Optimization, 990
- 20.2 Linear Programming, 994
- 20.3 Simplex Method, 998
- 20.4 Simplex Method: Degeneracy, Difficulties in Starting, 1002

Chapter Review, 1007
Chapter Summary, 1008

Chapter 21 Graphs and Combinatorial Optimization 1010

- 21.1 Graphs and Digraphs, 1010
- 21.2 Shortest Path Problems. Complexity, 1015
- 21.3 Bellman's Optimality Principle. Dijkstra's Algorithm, 1020
- 21.4 Shortest Spanning Trees. Kruskal's Greedy Algorithm, 1024

21.5 Prim's Algorithm for Shortest Spanning Trees, 1028
 21.6 Networks. Flow Augmenting Paths, 1031
 21.7 Ford–Fulkerson Algorithm for Maximum Flow, 1038
 21.8 Assignment Problems. Bipartite Matching, 1041
Chapter Review, 1046
Chapter Summary, 1048

Part G. Probability and Statistics 1049

Chapter 22 Data Analysis. Probability Theory 1050
 22.1 Data: Representation, Average, Spread, 1050
 22.2 Experiments, Outcomes, Events, 1055
 22.3 Probability, 1058
 22.4 Permutations and Combinations, 1064
 22.5 Random Variables, Probability Distributions, 1069
 22.6 Mean and Variance of a Distribution, 1075
 22.7 Binomial, Poisson, and Hypergeometric Distributions, 1079
 22.8 Normal Distribution, 1085
 22.9 Distributions of Several Random Variables, 1091
Chapter Review, 1100
Chapter Summary, 1102

Chapter 23 Mathematical Statistics. 1104
 23.1 Introduction. Random Sampling, 1104
 23.2 Estimation of Parameters, 1106
 23.3 Confidence Intervals, 1109
 23.4 Testing of Hypotheses, Decisions, 1118
 23.5 Quality Control, 1128
 23.6 Acceptance Sampling, 1133
 23.7 Goodness of Fit. χ^2 -Test, 1137
 23.8 Nonparametric Tests, 1142
 23.9 Regression Analysis. Fitting Straight Lines, 1145
 23.10 Correlation Analysis, 1150
Chapter Review, 1153
Chapter Summary, 1155

Appendix 1 References A1
Appendix 2 Answers to Odd-Numbered Problems A5
Appendix 3 Auxiliary Material A51

A3.1 Formulas for Special Functions, A51
 A3.2 Partial Derivatives, A57
 A3.3 Sequences and Series, A60

Appendix 4 Additional Proofs A65
Appendix 5 Tables A85
Index I1