A First Course in Discrete Mathematics

With 63 Figures
## Contents

1. Counting and Binomial Coefficients ............................. 1  
   1.1 Basic Principles ........................................ 1  
   1.2 Factorials ............................................. 2  
   1.3 Selections ............................................. 3  
   1.4 Binomial Coefficients and Pascal’s Triangle .............. 6  
   1.5 Selections with Repetitions .............................. 10  
   1.6 A Useful Matrix Inversion ................................ 13  

2. Recurrence ................................................................ 19  
   2.1 Some Examples ............................................ 19  
   2.2 The Auxiliary Equation Method ............................ 23  
   2.3 Generating Functions ..................................... 26  
   2.4 Derangements ............................................. 28  
   2.5 Sorting Algorithms ....................................... 32  
   2.6 Catalan Numbers ......................................... 34  

3. Introduction to Graphs ........................................... 43  
   3.1 The Concept of a Graph ................................... 43  
   3.2 Paths in Graphs .......................................... 46  
   3.3 Trees ..................................................... 47  
   3.4 Spanning Trees .......................................... 50  
   3.5 Bipartite Graphs ......................................... 52  
   3.6 Planarity .................................................. 54  
   3.7 Polyhedra ................................................. 60  

4. Travelling Round a Graph ....................................... 69  
   4.1 Hamiltonian Graphs ....................................... 69  
   4.2 Planarity and Hamiltonian Graphs ......................... 71  
   4.3 The Travelling Salesman Problem ......................... 74
4.4 Gray Codes .......................................................... 76
4.5 Eulerian Graphs ...................................................... 78
4.6 Eulerian Digraphs ................................................... 81

5. Partitions and Colourings ........................................ 89
  5.1 Partitions of a Set .................................................. 89
  5.2 Stirling Numbers .................................................. 91
  5.3 Counting Functions ............................................... 94
  5.4 Vertex Colourings of Graphs .................................... 96
  5.5 Edge Colourings of Graphs ....................................... 99

6. The Inclusion—Exclusion Principle ............................ 107
  6.1 The Principle ..................................................... 107
  6.2 Counting Surjections ............................................ 112
  6.3 Counting Labelled Trees ......................................... 113
  6.4 Scrabble .......................................................... 114
  6.5 The Ménage Problem .............................................. 115

7. Latin Squares and Hall’s Theorem .............................. 121
  7.1 Latin Squares and Orthogonality ............................... 121
  7.2 Magic Squares ................................................... 125
  7.3 Systems of Distinct Representatives .......................... 127
  7.4 From Latin Squares to Affine Planes .......................... 131

8. Schedules and 1-Factorisations ................................ 137
  8.1 The Circle Method ............................................... 137
  8.2 Bipartite Tournaments and 1-Factorisations of $K_{n,n}$ .... 142
  8.3 Tournaments from Orthogonal Latin Squares ................. 145

9. Introduction to Designs .......................................... 149
  9.1 Balanced Incomplete Block Designs ........................... 149
  9.2 Resolvable Designs ............................................... 156
  9.3 Finite Projective Planes ......................................... 159
  9.4 Hadamard Matrices and Designs ................................ 161
  9.5 Difference Methods ............................................... 165
  9.6 Hadamard Matrices and Codes ................................... 167

Appendix ................................................................. 179

Solutions ............................................................... 183

Further Reading ....................................................... 195

Bibliography .......................................................... 197

Index ............................................................... 199