STUDIES IN LOGIC
AND
THE FOUNDATIONS OF
MATHEMATICS

L. E. J. BROUWER
E. W. BETH
A. HEYTING

Editors

1958
NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM
# CONTENTS

Preface ...................................................................... v

## PART I. HISTORICAL INTRODUCTION

1. Introductory Remarks ................................................. 3
3. "Constructive" Axioms of "General" Set Theory ............... 9
4. The Axiom of Choice ............................................... 15
5. Axioms of Infinity and of Restriction ............................ 21
6. Development of Set-Theory from the Axioms of Z ........... 26
7. Remarks on the Axiom Systems of von Neumann, Bernays, Gödel ................................................. 31

## PART II. AXIOMATIC SET THEORY

Introduction .................................................................. 39

Chapter I. The Frame of Logic and Class Theory .......... 45
  1. Predicate Calculus; Class Terms and Descriptions; Explicit Definitions ................................................. 45
  2. Equality and Extensionality. Application to Descriptions .......................................................... 52
  3. Class Formalism. Class Operations .................................................. 56
  4. Functionality and Mappings .................................................. 61

Chapter II. The Start of General Set Theory .............. 65
  1. The Axioms of General Set Theory ................................. 65
  2. Aussonderungstheorem. Intersection ............................. 69
  3. Sum Theorem. Theorem of Replacement ........................ 72
  4. Functional Sets. One-to-one Correspondences .................. 76

Chapter III. Ordinals; Natural Numbers; Finite Sets ... 80
  1. Fundaments of the Theory of Ordinals ......................... 80
  2. Existential Statements on Ordinals. Limit Numbers ........ 86
  3. Fundaments of Number Theory .................................. 89
  4. Iteration. Primitive Recursion .................................... 92
  5. Finite Sets and Classes ............................................. 97

Chapter IV. Transfinite Recursion .......................... 100
  1. The General Recursion Theorem ................................. 100
  2. The Schema of Transfinite Recursion ......................... 104
  3. Generated Numeration ............................................. 109
CHAPTER V. POWER; ORDER; WELLORDER
1. Comparison of Powers
2. Order and Partial Order
3. Wellorder

CHAPTER VI. THE COMPLETING AXIOMS
1. The Potency Axiom
2. The Axiom of Choice
3. The Numeration Theorem. First Concepts of Cardinal Arithmetic
4. Zorn's Lemma and Related Principles
5. Axiom of Infinity. Denumerability

CHAPTER VII. ANALYSIS; CARDINAL ARITHMETIC; ABSTRACT THEORIES
1. Theory of Real Numbers
2. Some Topics of Ordinal Arithmetic
3. Cardinal Operations
4. Formal Laws on Cardinals
5. Abstract Theories

CHAPTER VIII. FURTHER STRENGTHENING OF THE AXIOM SYSTEM
1. A Strengthening of the Axiom of Choice
2. The Fundierungsaxiom
3. A one-to-one Correspondence between the Class of Ordinals and the Class of all Sets

INDEX OF AUTHORS (PART I)

INDEX OF SYMBOLS (PART II)
Predicates
Functors and Operators
Primitive Symbols

INDEX OF MATTERS (PART II)

LIST OF AXIOMS (PART II)

BIBLIOGRAPHY (PART I AND II)