Weston M. Stacey

Nuclear Reactor Physics

Second Edition, Completely Revised and Enlarged

WILEY-VCH Verlag GmbH & Co. KGaA
## Contents

Preface  xxiii  
Preface to 2nd Edition  xxvii

**PART 1  BASIC REACTOR PHYSICS**

1  Neutron Nuclear Reactions  3  
1.1 Neutron-Induced Nuclear Fission  3  
   Stable Nuclides  3  
   Binding Energy  3  
   Threshold External Energy for Fission  4  
   Neutron-Induced Fission  5  
   Neutron Fission Cross Sections  5  
   Products of the Fission Reaction  8  
   Energy Release  10  
1.2 Neutron Capture  13  
   Radiative Capture  13  
   Neutron Emission  19  
1.3 Neutron Elastic Scattering  20  
1.4 Summary of Cross-Section Data  24  
   Low-Energy Cross Sections  24  
   Spectrum-Averaged Cross Sections  24  
1.5 Evaluated Nuclear Data Files  24  
1.6 Elastic Scattering Kinematics  27  
   Correlation of Scattering Angle and Energy Loss  28  
   Average Energy Loss  29

2  Neutron Chain Fission Reactors  33  
2.1 Neutron Chain Fission Reactions  33  
   Capture-to-Fission Ratio  33  
   Number of Fission Neutrons per Neutron Absorbed in Fuel  33
3 Neutron Diffusion Theory

3.1 Derivation of One-Speed Diffusion Theory
Partial and Net Currents
Diffusion Theory
Interface Conditions
Boundary Conditions
Applicability of Diffusion Theory

3.2 Solutions of the Neutron Diffusion Equation in Nonmultiplying Media
Plane Isotropic Source in an Infinite Homogeneous Medium
Plane Isotropic Source in a Finite Homogeneous Medium
Line Source in an Infinite Homogeneous Medium
Homogeneous Cylinder of Infinite Axial Extent with Axial Line Source
Point Source in an Infinite Homogeneous Medium
Point Source at the Center of a Finite Homogeneous Sphere

3.3 Diffusion Kernels and Distributed Sources in a Homogeneous Medium
Infinite-Medium Diffusion Kernels
Finite-Slab Diffusion Kernel
Finite Slab with Incident Neutron Beam

3.4 Albedo Boundary Condition

3.5 Neutron Diffusion and Migration Lengths
Thermal Diffusion-Length Experiment
Migration Length

3.6 Bare Homogeneous Reactor
Slab Reactor
Right Circular Cylinder Reactor
Interpretation of Criticality Condition 60
Optimum Geometries 61

3.7 Reflected Reactor 62
Reflected Slab Reactor 62
Reflector Savings 64
Reflected Spherical, Cylindrical, and Rectangular Parallelepiped Cores 65

3.8 Homogenization of a Heterogeneous Fuel-Moderator Assembly 65
Spatial Self-Shielding and Thermal Disadvantage Factor 65
Effective Homogeneous Cross Sections 69
Thermal Utilization 71
Measurement of Thermal Utilization 72
Local Power Peaking Factor 73

3.9 Control Rods 73
Effective Diffusion Theory Cross Sections for Control Rods 73
Windowshade Treatment of Control Rods 76

3.10 Numerical Solution of Diffusion Equation 77
Finite Difference Equations in One Dimension 78
Forward Elimination/Backward Substitution Spatial Solution Procedure 79
Power Iteration on Fission Source 79
Finite-Difference Equations in Two Dimensions 80
Successive Relaxation Solution of Two-Dimensional Finite-Difference Equations 82
Power Outer Iteration on Fission Source 82
Limitations on Mesh Spacing 83

3.11 Nodal Approximation 83

3.12 Transport Methods 85
Transmission and Absorption in a Purely Absorbing Slab Control Plate 87
Escape Probability in a Slab 87
Integral Transport Formulation 87
Collision Probability Method 88
Differential Transport Formulation 89
Spherical Harmonics Methods 90
Discrete Ordinates Method 94

4 Neutron Energy Distribution 101
4.1 Analytical Solutions in an Infinite Medium 101
Fission Source Energy Range 102
Slowing-Down Energy Range 102
Moderation by Hydrogen Only 103
Energy Self-Shielding 103
Slowing Down by Nonhydrogenic Moderators with No Absorption 104
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>Delayed Neutron Kernel and Zero-Power Transfer Function</td>
<td>155</td>
</tr>
<tr>
<td></td>
<td>Delayed Neutron Kernel</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Zero-Power Transfer Function</td>
<td></td>
</tr>
<tr>
<td>5.6</td>
<td>Experimental Determination of Neutron Kinetics Parameters</td>
<td>156</td>
</tr>
<tr>
<td></td>
<td>Asymptotic Period Measurement</td>
<td>156</td>
</tr>
<tr>
<td></td>
<td>Rod Drop Method</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td>Source Jerk Method</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td>Pulsed Neutron Methods</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td>Rod Oscillator Measurements</td>
<td>158</td>
</tr>
<tr>
<td></td>
<td>Zero-Power Transfer Function Measurements</td>
<td>159</td>
</tr>
<tr>
<td></td>
<td>Rossi-α Measurement</td>
<td>159</td>
</tr>
<tr>
<td>5.7</td>
<td>Reactivity Feedback</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>Temperature Coefficients of Reactivity</td>
<td>162</td>
</tr>
<tr>
<td></td>
<td>Doppler Effect</td>
<td>162</td>
</tr>
<tr>
<td></td>
<td>Fuel and Moderator Expansion Effect on Resonance Escape Probability</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>Thermal Utilization</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td>Nonleakage Probability</td>
<td>166</td>
</tr>
<tr>
<td></td>
<td>Representative Thermal Reactor Reactivity Coefficients</td>
<td>166</td>
</tr>
<tr>
<td></td>
<td>Startup Temperature Defect</td>
<td>167</td>
</tr>
<tr>
<td>5.8</td>
<td>Perturbation Theory Evaluation of Reactivity Temperature Coefficients</td>
<td>168</td>
</tr>
<tr>
<td></td>
<td>Perturbation Theory</td>
<td>168</td>
</tr>
<tr>
<td></td>
<td>Sodium Void Effect in Fast Reactors</td>
<td>169</td>
</tr>
<tr>
<td></td>
<td>Doppler Effect in Fast Reactors</td>
<td>169</td>
</tr>
<tr>
<td></td>
<td>Fuel and Structure Motion in Fast Reactors</td>
<td>170</td>
</tr>
<tr>
<td></td>
<td>Fuel Bowing</td>
<td>171</td>
</tr>
<tr>
<td></td>
<td>Representative Fast Reactor Reactivity Coefficients</td>
<td>171</td>
</tr>
<tr>
<td>5.9</td>
<td>Reactor Stability</td>
<td>171</td>
</tr>
<tr>
<td></td>
<td>Reactor Transfer Function with Reactivity Feedback</td>
<td>171</td>
</tr>
<tr>
<td></td>
<td>Stability Analysis for a Simple Feedback Model</td>
<td>172</td>
</tr>
<tr>
<td></td>
<td>Threshold Power Level for Reactor Stability</td>
<td>174</td>
</tr>
<tr>
<td></td>
<td>More General Stability Conditions</td>
<td>175</td>
</tr>
<tr>
<td></td>
<td>Power Coefficients and Feedback Delay Time Constants</td>
<td>178</td>
</tr>
<tr>
<td>5.10</td>
<td>Measurement of Reactor Transfer Functions</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>Rod Oscillator Method</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>Correlation Methods</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>Reactor Noise Method</td>
<td>181</td>
</tr>
<tr>
<td>5.11</td>
<td>Reactor Transients with Feedback</td>
<td>183</td>
</tr>
<tr>
<td></td>
<td>Step Reactivity Insertion (ρ_{ex} &lt; β): Prompt Jump</td>
<td>184</td>
</tr>
<tr>
<td></td>
<td>Step Reactivity Insertion (ρ_{ex} &lt; β): Post-Prompt-Jump Transient</td>
<td>185</td>
</tr>
<tr>
<td>5.12</td>
<td>Reactor Fast Excursions</td>
<td>186</td>
</tr>
<tr>
<td></td>
<td>Step Reactivity Input: Feedback Proportional to Fission Energy</td>
<td>186</td>
</tr>
<tr>
<td></td>
<td>Ramp Reactivity Input: Feedback Proportional to Fission Energy</td>
<td>187</td>
</tr>
</tbody>
</table>
Step Reactivity Input: Nonlinear Feedback Proportional to Cumulative Energy Release 187
Bethe–Tait Model 188

5.13 Numerical Methods 190

### 6 Fuel Burnup 197

6.1 Changes in Fuel Composition 197
   Fuel Transmutation–Decay Chains 198
   Fuel Depletion–Transmutation–Decay Equations 199
   Fission Products 203
   Solution of the Depletion Equations 204
   Measure of Fuel Burnup 205
   Fuel Composition Changes with Burnup 205
   Reactivity Effects of Fuel Composition Changes 206
   Compensating for Fuel-Depletion Reactivity Effects 208
   Reactivity Penalty 208
   Effects of Fuel Depletion on the Power Distribution 209
   In-Core Fuel Management 210

6.2 Samarium and Xenon 211
   Samarium Poisoning 211
   Xenon Poisoning 213
   Peak Xenon 215
   Effect of Power-Level Changes 216

6.3 Fertile-to-Fissile Conversion and Breeding 217
   Availability of Neutrons 217
   Conversion and Breeding Ratios 219

6.4 Simple Model of Fuel Depletion 219

6.5 Fuel Reprocessing and Recycling 221
   Composition of Recycled LWR Fuel 221
   Physics Differences of MOX Cores 222
   Physics Considerations with Uranium Recycle 224
   Physics Considerations with Plutonium Recycle 225
   Reactor Fueling Characteristics 225

6.6 Radioactive Waste 226
   Radioactivity 226
   Hazard Potential 226
   Risk Factor 226

6.7 Burning Surplus Weapons-Grade Uranium and Plutonium 233
   Composition of Weapons-Grade Uranium and Plutonium 233
   Physics Differences Between Weapons- and Reactor-Grade Plutonium-Fueled Reactors 234

6.8 Utilization of Uranium Energy Content 235

6.9 Transmutation of Spent Nuclear Fuel 237

6.10 Closing the Nuclear Fuel Cycle 244
7 Nuclear Power Reactors 249

7.1 Pressurized Water Reactors 249
7.2 Boiling Water Reactors 250
7.3 Pressure Tube Heavy Water–Moderated Reactors 255
7.4 Pressure Tube Graphite-Moderated Reactors 258
7.5 Graphite-Moderated Gas-Cooled Reactors 260
7.6 Liquid-Metal Fast Breeder Reactors 261
7.7 Other Power Reactors 265
7.8 Characteristics of Power Reactors 265
7.9 Advanced Generation-III Reactors 265
   Advanced Boiling Water Reactors (ABWR) 266
   Advanced Pressurized Water Reactors (APWR) 267
   Advanced Pressure Tube Reactor 268
   Modular High-Temperature Gas-Cooled Reactors (GT-MHR) 268
7.10 Advanced Generation-IV Reactors 269
   Gas-Cooled Fast Reactors (GFR) 270
   Lead-Cooled Fast Reactors (LFR) 271
   Molten Salt Reactors (MSR) 271
   Super-Critical Water Reactors (SCWR) 272
   Sodium-Cooled Fast Reactors (SFR) 272
   Very High Temperature Reactors (VHTR) 272
7.11 Advanced Sub-critical Reactors 273
7.12 Nuclear Reactor Analysis 275
   Construction of Homogenized Multigroup Cross Sections 275
   Criticality and Flux Distribution Calculations 276
   Fuel Cycle Analyses 277
   Transient Analyses 278
   Core Operating Data 279
   Criticality Safety Analysis 279
7.13 Interaction of Reactor Physics and Reactor Thermal Hydraulics 280
   Power Distribution 280
   Temperature Reactivity Effects 281
   Coupled Reactor Physics and Thermal-Hydraulics Calculations 281

8 Reactor Safety 283

8.1 Elements of Reactor Safety 283
   Radionuclides of Greatest Concern 283
   Multiple Barriers to Radionuclide Release 283
   Defense in Depth 285
   Energy Sources 285
8.2 Reactor Safety Analysis 285
   Loss of Flow or Loss of Coolant 287
   Loss of Heat Sink 287


<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactivity Insertion</td>
<td>287</td>
</tr>
<tr>
<td>Anticipated Transients without Scram</td>
<td>288</td>
</tr>
<tr>
<td>8.3 Quantitative Risk Assessment</td>
<td>288</td>
</tr>
<tr>
<td>Probabilistic Risk Assessment</td>
<td>288</td>
</tr>
<tr>
<td>Radiological Assessment</td>
<td>291</td>
</tr>
<tr>
<td>Reactor Risks</td>
<td>291</td>
</tr>
<tr>
<td>8.4 Reactor Accidents</td>
<td>293</td>
</tr>
<tr>
<td>Three Mile Island</td>
<td>294</td>
</tr>
<tr>
<td>Chernobyl</td>
<td>297</td>
</tr>
<tr>
<td>8.5 Passive Safety</td>
<td>299</td>
</tr>
<tr>
<td>Pressurized Water Reactors</td>
<td>299</td>
</tr>
<tr>
<td>Boiling Water Reactors</td>
<td>299</td>
</tr>
<tr>
<td>Integral Fast Reactors</td>
<td>300</td>
</tr>
<tr>
<td>Passive Safety Demonstration</td>
<td>300</td>
</tr>
</tbody>
</table>

**PART 2 ADVANCED REACTOR PHYSICS**

9 Neutron Transport Theory 305

9.1 Neutron Transport Equation 305

*Boundary Conditions* 310

*Scalar Flux and Current* 310

*Partial Currents* 310

9.2 Integral Transport Theory 310

*Isotropic Point Source* 311

*Isotropic Plane Source* 311

*Anisotropic Plane Source* 312

*Transmission and Absorption Probabilities* 314

*Escape Probability* 314

*First-Collision Source for Diffusion Theory* 315

*Inclusion of Isotropic Scattering and Fission* 315

*Distributed Volumetric Sources in Arbitrary Geometry* 316

*Flux from a Line Isotropic Source of Neutrons* 317

*Bickley Functions* 318

*Probability of Reaching a Distance t from a Line Isotropic Source without a Collision* 318

9.3 Collision Probability Methods 319

*Reciprocity Among Transmission and Collision Probabilities* 320

*Collision Probabilities for Slab Geometry* 320

*Collision Probabilities in Two-Dimensional Geometry* 321

*Collision Probabilities for Annular Geometry* 322

9.4 Interface Current Methods in Slab Geometry 323

*Emergent Currents and Reaction Rates Due to Incident Currents* 323

*Emergent Currents and Reaction Rates Due to Internal Sources* 326
Total Reaction Rates and Emergent Currents 327
Boundary Conditions 329
Response Matrix 329

9.5 Multidimensional Interface Current Methods 330
Extension to Multidimension 330
Evaluation of Transmission and Escape Probabilities 332
Transmission Probabilities in Two-Dimensional Geometries 333
Escape Probabilities in Two-Dimensional Geometries 335
Simple Approximations for the Escape Probability 337

9.6 Spherical Harmonics \((P_L)\) Methods in One-Dimensional Geometries 338
Legendre Polynomials 338
Neutron Transport Equation in Slab Geometry 339
\(P_L\) Equations 339
Boundary and Interface Conditions 340
\(P_1\) Equations and Diffusion Theory 342
Simplified \(P_L\) or Extended Diffusion Theory 343
\(P_L\) Equations in Spherical and Cylindrical Geometries 344
Diffusion Equations in One-Dimensional Geometry 347
Half-Angle Legendre Polynomials 347
Double-\(P_L\) Theory 348
D-\(P_0\) Equations 349

9.7 Multidimensional Spherical Harmonics \((P_L)\) Transport Theory 350
Spherical Harmonics 350
Spherical Harmonics Transport Equations in Cartesian Coordinates 351
\(P_1\) Equations in Cartesian Geometry 352
Diffusion Theory 353

9.8 Discrete Ordinates Methods in One-Dimensional Slab Geometry 354
\(P_L\) and D-\(P_L\) Ordinates 355
Spatial Differencing and Iterative Solution 357
Limitations on Spatial Mesh Size 358

9.9 Discrete Ordinates Methods in One-Dimensional Spherical Geometry 359
Representation of Angular Derivative 360
Iterative Solution Procedure 360
Acceleration of Convergence 362
Calculation of Criticality 362

9.10 Multidimensional Discrete Ordinates Methods 363
Ordinates and Quadrature Sets 363
\(S_N\) Method in Two-Dimensional \(x-y\) Geometry 366
Further Discussion 369

9.11 Even-Parity Transport Formulation 369

9.12 Monte Carlo Methods 371
Probability Distribution Functions 371
10 Neutron Slowing Down 385
10.1 Elastic Scattering Transfer Function 385
Lethargy 385
Elastic Scattering Kinematics 385
Elastic Scattering Kernel 386
Isotropic Scattering in Center-of-Mass System 388
Linearly Anisotropic Scattering in Center-of-Mass System 389
10.2 $P_1$ and $B_1$ Slowing-Down Equations 390
Derivation 390
Solution in Finite Uniform Medium 393
$B_1$ Equations 394
Few-Group Constants 395
10.3 Diffusion Theory 396
Lethargy-Dependent Diffusion Theory 396
Directional Diffusion Theory 397
Multigroup Diffusion Theory 398
Boundary and Interface Conditions 399
10.4 Continuous Slowing-Down Theory 400
$P_1$ Equations in Slowing-Down Density Formulation 400
Slowing-Down Density in Hydrogen 403
Heavy Mass Scatterers 404
Age Approximation 404
Selengut-Goertzel Approximation 405
Consistent $P_1$ Approximation 405
Extended Age Approximation 405
Grueling-Goertzel Approximation 406
Summary of $P_1$ Continuous Slowing-Down Theory 407
Inclusion of Anisotropic Scattering 407
Inclusion of Scattering Resonances 409
$P_1$ Continuous Slowing-Down Equations 410
10.5 Multigroup Discrete Ordinates Transport Theory 411

11 Resonance Absorption 415
11.1 Resonance Cross Sections 415
11.2 Widely Spaced Single-Level Resonances in a Heterogeneous Fuel–Moderator Lattice 415
Neutron Balance in Heterogeneous Fuel–Moderator Cell 415
Reciprocity Relation 418
Narrow Resonance Approximation 419
Wide Resonance Approximation 420
Evaluation of Resonance Integrals 420
Infinite Dilution Resonance Integral 422
Equivalence Relations 422
Heterogeneous Resonance Escape Probability 423
Homogenized Multigroup Resonance Cross Section 423
Improved and Intermediate Resonance Approximations 424
11.3 Calculation of First-Flight Escape Probabilities 424
Escape Probability for an Isolated Fuel Rod 425
Closely Packed Lattices 427
11.4 Unresolved Resonances 428
Multigroup Cross Sections for Isolated Resonances 430
Self-Overlap Effects 431
Overlap Effects for Different Sequences 432
11.5 Multiband Treatment of Spatially Dependent Self-Shielding 433
Spatially Dependent Self-Shielding 433
Multiband Theory 434
Evaluation of Multiband Parameters 436
Calculation of Multiband Parameters 437
Interface Conditions 439
11.6 Resonance Cross-Section Representations 439
R-Matrix Representation 439
Practical Formulations 441
Generalization of the Pole Representation 445
Doppler Broadening of the Generalized Pole Representation 448

12 Neutron Thermalization 453
12.1 Double Differential Scattering Cross Section for Thermal Neutrons 453
12.2 Neutron Scattering from a Monatomic Maxwellian Gas 454
Differential Scattering Cross Section 454
Cold Target Limit 455
Free-Hydrogen (Proton) Gas Model 455
Radkowsky Model for Scattering from H₂O 455
Heavy Gas Model 456
12.3 Thermal Neutron Scattering from Bound Nuclei 457
Pair Distribution Functions and Scattering Functions 457
Intermediate Scattering Functions 458
Incoherent Approximation 459
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian Representation of Scattering</td>
<td>459</td>
</tr>
<tr>
<td>Measurement of the Scattering Function</td>
<td>460</td>
</tr>
<tr>
<td>Applications to Neutron Moderating Media</td>
<td>460</td>
</tr>
<tr>
<td>12.4 Calculation of the Thermal Neutron Spectra in Homogeneous Media</td>
<td>462</td>
</tr>
<tr>
<td>Wigner–Wilkins Proton Gas Model</td>
<td>463</td>
</tr>
<tr>
<td>Heavy Gas Model</td>
<td>466</td>
</tr>
<tr>
<td>Numerical Solution</td>
<td>468</td>
</tr>
<tr>
<td>Moments Expansion Solution</td>
<td>470</td>
</tr>
<tr>
<td>Multigroup Calculation</td>
<td>473</td>
</tr>
<tr>
<td>Applications to Moderators</td>
<td>474</td>
</tr>
<tr>
<td>12.5 Calculation of Thermal Neutron Energy Spectra in Heterogeneous Lattices</td>
<td>474</td>
</tr>
<tr>
<td>12.6 Pulsed Neutron Thermalization</td>
<td>477</td>
</tr>
<tr>
<td>Spatial Eigenfunction Expansion</td>
<td>477</td>
</tr>
<tr>
<td>Energy Eigenfunctions of the Scattering Operator</td>
<td>477</td>
</tr>
<tr>
<td>Expansion in Energy Eigenfunctions of the Scattering Operator</td>
<td>479</td>
</tr>
<tr>
<td>13 <strong>Perturbation and Variational Methods</strong></td>
<td>483</td>
</tr>
<tr>
<td>13.1 Perturbation Theory Reactivity Estimate</td>
<td>483</td>
</tr>
<tr>
<td>Multigroup Diffusion Perturbation Theory</td>
<td>483</td>
</tr>
<tr>
<td>13.2 Adjoint Operators and Importance Function</td>
<td>486</td>
</tr>
<tr>
<td>Adjoint Operators</td>
<td>486</td>
</tr>
<tr>
<td>Importance Interpretation of the Adjoint Function</td>
<td>487</td>
</tr>
<tr>
<td>Eigenvalues of the Adjoint Equation</td>
<td>489</td>
</tr>
<tr>
<td>13.3 Variational/Generalized Perturbation Reactivity Estimate</td>
<td>489</td>
</tr>
<tr>
<td>One-Speed Diffusion Theory</td>
<td>490</td>
</tr>
<tr>
<td>Other Transport Models</td>
<td>493</td>
</tr>
<tr>
<td>Reactivity Worth of Localized Perturbations in a Large PWR Core Model</td>
<td>494</td>
</tr>
<tr>
<td>Higher-Order Variational Estimates</td>
<td>495</td>
</tr>
<tr>
<td>13.4 Variational/Generalized Perturbation Theory Estimates of Reaction Rate Ratios in Critical Reactors</td>
<td>495</td>
</tr>
<tr>
<td>13.5 Variational/Generalized Perturbation Theory Estimates of Reaction Rates</td>
<td>497</td>
</tr>
<tr>
<td>13.6 Variational Theory</td>
<td>498</td>
</tr>
<tr>
<td>Stationarity</td>
<td>498</td>
</tr>
<tr>
<td>Roussopolos Variational Functional</td>
<td>498</td>
</tr>
<tr>
<td>Schwinger Variational Functional</td>
<td>499</td>
</tr>
<tr>
<td>Rayleigh Quotient</td>
<td>499</td>
</tr>
<tr>
<td>Construction of Variational Functionals</td>
<td>500</td>
</tr>
<tr>
<td>13.7 Variational Estimate of Intermediate Resonance Integral</td>
<td>500</td>
</tr>
<tr>
<td>13.8 Heterogeneity Reactivity Effects</td>
<td>502</td>
</tr>
<tr>
<td>13.9 Variational Derivation of Approximate Equations</td>
<td>503</td>
</tr>
</tbody>
</table>
13.10 Variational Even-Parity Transport Approximations 505
Variational Principle for the Even-Parity Transport Equation 505
Ritz Procedure 506
Diffusion Approximation 507
One-Dimensional Slab Transport Equation 508
13.11 Boundary Perturbation Theory 508

14 Homogenization 515
14.1 Equivalent Homogenized Cross Sections 516
14.2 ABH Collision Probability Method 517
14.3 Blackness Theory 520
14.4 Fuel Assembly Transport Calculations 522
Pin Cells 522
Wigner–Seitz Approximation 523
Collision Probability Pin-Cell Model 524
Interface Current Formulation 527
Multigroup Pin-Cell Collision Probabilities Model 528
Resonance Cross Sections 529
Full Assembly Transport Calculation 529
14.5 Homogenization Theory 529
Homogenization Considerations 530
Conventional Homogenization Theory 531
14.6 Equivalence Homogenization Theory 531
14.7 Multiscale Expansion Homogenization Theory 535
14.8 Flux Detail Reconstruction 538

15 Nodal and Synthesis Methods 541
15.1 General Nodal Formalism 542
15.2 Conventional Nodal Methods 544
15.3 Transverse Integrated Nodal Diffusion Theory Methods 547
Transverse Integrated Equations 547
Polynomial Expansion Methods 549
Analytical Methods 553
Heterogeneous Flux Reconstruction 554
15.4 Transverse Integrated Nodal Integral Transport Theory Models 554
Transverse Integrated Integral Transport Equations 554
Polynomial Expansion of Scalar Flux 557
Isotropic Component of Transverse Leakage 558
Double-$P_n$ Expansion of Surface Fluxes 558
Angular Moments of Outgoing Surface Fluxes 559
Nodal Transport Equations 561
15.5 Transverse Integrated Nodal Discrete Ordinates Method 561
15.6 Finite-Element Coarse Mesh Methods 563
Contents

Variational Functional for the $P_1$ Equations 563
One-Dimensional Finite-Difference Approximation 564
Diffusion Theory Variational Functional 566
Linear Finite-Element Diffusion Approximation in One Dimension 567
Higher-Order Cubic Hermite Coarse-Mesh Diffusion Approximation 569
Multidimensional Finite-Element Coarse-Mesh Methods 570
15.7 Variational Discrete Ordinates Nodal Method 571
Variational Principle 571
Application of the Method 579
15.8 Variational Principle for Multigroup Diffusion Theory 580
15.9 Single-Channel Spatial Synthesis 583
15.10 Multichannel Spatial Synthesis 589
15.11 Spectral Synthesis 591

16 Space-Time Neutron Kinetics 599
16.1 Flux Tilts and Delayed Neutron Holdback 599
Modal Eigenfunction Expansion 600
Flux Tilts 601
Delayed Neutron Holdback 602
16.2 Spatially Dependent Point Kinetics 602
Derivation of Point Kinetics Equations 604
Adiabatic and Quasistatic Methods 605
Variational Principle for Static Reactivity 606
Variational Principle for Dynamic Reactivity 607
16.3 Time Integration of the Spatial Neutron Flux Distribution 609
Explicit Integration: Forward-Difference Method 610
Implicit Integration: Backward-Difference Method 611
Implicit Integration: $\theta$ Method 612
Implicit Integration: Time-Integrated Method 615
Implicit Integration: GAKIN Method 616
Alternating Direction Implicit Method 619
Stiffness Confinement Method 622
Symmetric Successive Overrelaxation Method 623
Generalized Runge–Kutta Methods 624
16.4 Stability 625
Classical Linear Stability Analysis 625
Lyapunov's Method 627
Lyapunov's Method for Distributed Parameter Systems 629
Control 631
Variational Methods of Control Theory 631
Dynamic Programming 633
Pontryagin's Maximum Principle 634
Variational Methods for Spatially Dependent Control Problems 636
Dynamic Programming for Spatially Continuous Systems 638
Pontryagin's Maximum Principle for a Spatially Continuous System 639

16.5 Xenon Spatial Oscillations 641
Linear Stability Analysis 642
μ-Mode Approximation 644
λ-Mode Approximation 645
Nonlinear Stability Criterion 649
Control of Xenon Spatial Power Oscillations 650
Variational Control Theory of Xenon Spatial Oscillations 650

16.6 Stochastic Kinetics 652
Forward Stochastic Model 653
Means, Variances, and Covariances 656
Correlation Functions 658
Physical Interpretation, Applications, and Initial and Boundary Conditions 659
Numerical Studies 660
Startup Analysis 663

APPENDICES

A Physical Constants and Nuclear Data 669
B Some Useful Mathematical Formulas 675
C Step Functions, Delta Functions, and Other Functions 677
C.1 Introduction 677
C.2 Properties of the Dirac δ-Function 678
A. Alternative Representations 678
B. Properties 678
C. Derivatives 679
D Some Properties of Special Functions 681
E Introduction to Matrices and Matrix Algebra 687
E.1 Some Definitions 687
E.2 Matrix Algebra 689
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction to Laplace Transforms</td>
<td>691</td>
</tr>
<tr>
<td>F.1 Motivation</td>
<td>691</td>
</tr>
<tr>
<td>F.2 &quot;Cookbook&quot; Laplace transforms</td>
<td>694</td>
</tr>
<tr>
<td>Index</td>
<td>697</td>
</tr>
</tbody>
</table>