Sandra Gruescu

Population Ageing and Economic Growth

Education Policy and Family Policy in a Model of Endogenous Growth

With 7 Figures and 32 Tables

Physica-Verlag
A Springer Company
# Contents

## Part I Population in models of economic growth

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Introduction</td>
<td>3</td>
</tr>
<tr>
<td>1.1 Motivation and main question of this research</td>
<td>3</td>
</tr>
<tr>
<td>1.2 Organization of the research</td>
<td>5</td>
</tr>
<tr>
<td>1.3 The ageing population: Trends in Germany, United Kingdom and the USA</td>
<td>6</td>
</tr>
<tr>
<td>2 The size and the growth rate of population and economic growth</td>
<td>13</td>
</tr>
<tr>
<td>2.1 General literature on ageing population, declining population size and declining population growth rate</td>
<td>13</td>
</tr>
<tr>
<td>2.2 Population in economic growth theory with exogenous technological progress</td>
<td>16</td>
</tr>
<tr>
<td>2.3 Population in economic growth theory with endogenous technological progress</td>
<td>19</td>
</tr>
<tr>
<td>2.3.1 Models with scale effects</td>
<td>20</td>
</tr>
<tr>
<td>2.3.2 Models without scale effects</td>
<td>25</td>
</tr>
<tr>
<td>2.3.3 Scale effects and policy intervention to increase economic growth</td>
<td>28</td>
</tr>
<tr>
<td>3 Effects of a declining population in a model of economic growth</td>
<td>31</td>
</tr>
<tr>
<td>3.1 Introduction to a model of economic growth - The Solow (1956) model without technological progress</td>
<td>32</td>
</tr>
<tr>
<td>3.1.1 The real growth rate</td>
<td>32</td>
</tr>
<tr>
<td>3.1.2 The steady state</td>
<td>37</td>
</tr>
<tr>
<td>3.1.3 Transitional dynamics</td>
<td>41</td>
</tr>
<tr>
<td>3.1.4 Golden consumption</td>
<td>42</td>
</tr>
<tr>
<td>3.1.5 Speed of convergence</td>
<td>44</td>
</tr>
</tbody>
</table>
X  Contents

3.1.6 Conclusions on population and economic growth in
the Solow model ........................................... 46

3.2 The Solow (1956) model with technological progress ........ 48
  3.2.1 The real growth rate .................................. 48
  3.2.2 The steady state ..................................... 50
  3.2.3 Transitional dynamics ................................. 51
  3.2.4 Golden consumption .................................. 52
  3.2.5 Speed of convergence ................................ 54
  3.2.6 Conclusions on population and economic growth in
the Solow model with technological progress ............... 55

3.3 A model of economic growth with human capital - Mankiw,
Romer and Weil (1992) ....................................... 57
  3.3.1 The real growth rate .................................. 57
  3.3.2 The steady state ..................................... 58
  3.3.3 Transitional dynamics ................................. 59
  3.3.4 Golden consumption .................................. 60
  3.3.5 Speed of convergence ................................ 61
  3.3.6 Conclusions on population and economic growth in
the Mankiw, Romer, Weil model ............................ 62

3.4 A model of economic growth with human capital and age
structure - Lindh and Malmberg (1999) ....................... 62
  3.4.1 The real growth rate .................................. 63
  3.4.2 The steady state ..................................... 63
  3.4.3 Transitional dynamics ................................. 64
  3.4.4 Golden consumption .................................. 65
  3.4.5 Speed of convergence ................................ 65
  3.4.6 Conclusions on population and economic growth in
the Lindh-Malmberg model ................................. 66

4 Effects of a declining population in a model of economic
growth with endogenous human capital - Lucas (1988) .... 67
  4.1 The problem of dynamic optimization
and its solution ............................................. 67
    4.1.1 The problem of dynamic optimization with multiple
control and state variables ................................ 70
  4.2 A model of economic growth with human capital - Lucas (1988) 72
    4.2.1 The real growth rate ................................ 73
    4.2.2 The household ...................................... 74
    4.2.3 The Hamiltonian approach ........................... 75
    4.2.4 The steady state ................................... 79
    4.2.5 Conclusions on population and economic growth in
the Lucas model ........................................... 81
  4.3 A note on the Lucas model ................................ 82

5 Conclusions of Part I ...................................... 89
Part II Models of economic growth with an ageing population

6 Models of “Silver Growth” ............................................................... 95

7 Models with exogenous population .............................................. 97
  7.1 Model 1: The Solow (1956) model with an ageing population .... 97
      7.1.1 The real growth rate ....................................................... 98
      7.1.2 The steady state ........................................................... 99
      7.1.3 Transitional dynamics ................................................... 102
      7.1.4 Silver consumption ...................................................... 103
      7.1.5 Speed of convergence .................................................. 105
  7.2 Model 2: The Lucas (1988) model with an ageing population .... 108
      7.2.1 The real growth rate ....................................................... 108
      7.2.2 The steady state ........................................................... 112
      7.2.3 Comparison of Model 2 and Lucas (1988) ......................... 115

8 Models with quasi-endogenous population ................................... 119
  8.1 Model 3: The Lucas model with the new time allocation and
      quasi-endogenous population growth ....................................... 121
      8.1.1 The real growth rate ....................................................... 123
      8.1.2 The steady state ........................................................... 129
      8.1.3 Comparison of Model 3 and Lucas (1988) ......................... 131
      8.1.4 A quantitative solution of Model 3 .................................. 132
  8.2 Model 4: A model of silver growth with the new time
      allocation, quasi-endogenous population growth and an
      ageing population ............................................................... 134
      8.2.1 The real growth rate ....................................................... 136
      8.2.2 The steady state ........................................................... 142
      8.2.3 Comparative statics ...................................................... 145
      8.2.4 Comparison of Model 3 and Model 4 ............................... 148

9 Models with endogenous population ........................................... 151
  9.1 Model 5: The Lucas model with the new time allocation and
      endogenous population growth ............................................. 151
      9.1.1 The real growth rate ....................................................... 154
      9.1.2 The steady state ........................................................... 157
      9.1.3 Comparative statics and transitional dynamics ................. 160
  9.2 Model 6: A model of silver growth with the new time
      allocation, endogenous population growth and an ageing
      population ................................................................. 163
      9.2.1 The real growth rate ....................................................... 164
      9.2.2 The steady state ........................................................... 168
      9.2.3 Comparative statics and transitional dynamics ................. 171