Fundamentals of Algebraic Graph Transformation

With 41 Figures
## Contents

### Part I Introduction to Graph Transformation Systems

1. **General Introduction** .................................................. 5
   1.1 General Overview of Graph Grammars and Graph Transformation .................................................. 5
     1.1.1 What Is Graph Transformation? ......................... 6
     1.1.2 Aims and Paradigms of Graph Transformation .......... 6
     1.1.3 Overview of Various Approaches ....................... 9
   1.2 The Main Ideas of the Algebraic Graph Transformation Approach .................................................. 10
     1.2.1 The DPO Approach ........................................... 11
     1.2.2 The Algebraic Roots ....................................... 12
     1.2.3 From the DPO to the SPO Approach .................... 13
     1.2.4 From Graphs to High-Level Structures ................. 14
   1.3 The Chapters of This Book and the Main Results .......... 15
     1.3.1 Part I: Introduction to Graph Transformation Systems .................. 15
     1.3.2 Part II: Adhesive HLR Categories and Systems ....... 15
     1.3.3 Part III: Typed Attributed Graph Transformation Systems .................................................. 16
     1.3.4 Part IV: Case Study and Tool Support ................. 17
     1.3.5 Appendices .................................................. 17
     1.3.6 Hints for Reading This Book ............................. 17
   1.4 Bibliographic Notes and Further Topics .................... 17
     1.4.1 Concepts of Graph Grammars and Graph Transformation Systems .................................................. 17
     1.4.2 Application Areas of Graph Transformation Systems ... 19
     1.4.3 Languages and Tools for Graph Transformation Systems 19
     1.4.4 Future Work ................................................. 20
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Graphs, Typed Graphs, and the Gluing Construction</td>
<td>21</td>
</tr>
<tr>
<td>2.1</td>
<td>Graphs and Typed Graphs</td>
<td>21</td>
</tr>
<tr>
<td>2.2</td>
<td>Introduction to Categories</td>
<td>25</td>
</tr>
<tr>
<td>2.3</td>
<td>Pushouts as a Gluing Construction</td>
<td>29</td>
</tr>
<tr>
<td>2.4</td>
<td>Pullbacks as the Dual Construction of Pushouts</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>Graph Transformation Systems</td>
<td>37</td>
</tr>
<tr>
<td>3.1</td>
<td>Basic Definitions for GT Systems</td>
<td>37</td>
</tr>
<tr>
<td>3.2</td>
<td>Construction of Graph Transformations</td>
<td>43</td>
</tr>
<tr>
<td>3.3</td>
<td>Local Church–Rosser and Parallelism Theorems for GT Systems</td>
<td>47</td>
</tr>
<tr>
<td>3.4</td>
<td>Overview of Some Other Main Results for GT Systems</td>
<td>53</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Concurrency Theorem</td>
<td>54</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Embedding and Extension Theorems</td>
<td>56</td>
</tr>
<tr>
<td>3.4.3</td>
<td>Confluence, Local Confluence, Termination, and Critical Pairs</td>
<td>59</td>
</tr>
<tr>
<td>3.4.4</td>
<td>Functional Behavior of GT Systems and Termination Analysis</td>
<td>62</td>
</tr>
<tr>
<td>3.5</td>
<td>Graph Constraints and Application Conditions</td>
<td>64</td>
</tr>
</tbody>
</table>

Part II Adhesive High-Level Replacement Categories and Systems

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Adhesive High-Level Replacement Categories</td>
<td>77</td>
</tr>
<tr>
<td>4.1</td>
<td>Van Kampen Squares and Adhesive Categories</td>
<td>77</td>
</tr>
<tr>
<td>4.2</td>
<td>Adhesive HLR Categories</td>
<td>86</td>
</tr>
<tr>
<td>4.3</td>
<td>HLR Properties of Adhesive HLR Categories</td>
<td>96</td>
</tr>
<tr>
<td>5</td>
<td>Adhesive High-Level Replacement Systems</td>
<td>101</td>
</tr>
<tr>
<td>5.1</td>
<td>Basic Concepts of Adhesive HLR Systems</td>
<td>101</td>
</tr>
<tr>
<td>5.2</td>
<td>Instantiation of Adhesive HLR Systems</td>
<td>105</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Graph and Typed Graph Transformation Systems</td>
<td>106</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Hypergraph Transformation Systems</td>
<td>106</td>
</tr>
<tr>
<td>5.2.3</td>
<td>Petri Net Transformation Systems</td>
<td>107</td>
</tr>
<tr>
<td>5.2.4</td>
<td>Algebraic Specification Transformation Systems</td>
<td>108</td>
</tr>
<tr>
<td>5.2.5</td>
<td>Typed Attributed Graph Transformation Systems</td>
<td>108</td>
</tr>
<tr>
<td>5.3</td>
<td>The Local Church–Rosser and Parallelism Theorems</td>
<td>109</td>
</tr>
<tr>
<td>5.4</td>
<td>Concurrency Theorem and Pair Factorization</td>
<td>117</td>
</tr>
<tr>
<td>6</td>
<td>Embedding and Local Confluence</td>
<td>125</td>
</tr>
<tr>
<td>6.1</td>
<td>Initial Pushouts and the Gluing Condition</td>
<td>125</td>
</tr>
<tr>
<td>6.2</td>
<td>Embedding and Extension Theorems</td>
<td>130</td>
</tr>
<tr>
<td>6.3</td>
<td>Critical Pairs</td>
<td>140</td>
</tr>
<tr>
<td>6.4</td>
<td>Local Confluence Theorem</td>
<td>144</td>
</tr>
</tbody>
</table>
13 Typed Attributed Graph Transformation with Inheritance 259
  13.1 Attributed Type Graphs with Inheritance 260
  13.2 Attributed Clan Morphisms 265
  13.3 Productions and Attributed Graph Transformation with Inheritance 271
  13.4 Equivalence of Concepts with and without Inheritance 278

Part IV Case Study on Model Transformation, and Tool Support by AGG

14 Case Study on Model Transformation 287
  14.1 Model Transformation by Typed Attributed Graph Transformation 287
  14.2 Model Transformation from Statecharts to Petri Nets 288
    14.2.1 Source Modeling Language: Simple Version of UML Statecharts 289
    14.2.2 Target Modeling Language: Petri Nets 290
    14.2.3 Model Transformation 293
    14.2.4 Termination Analysis of the Model Transformation 301
  14.3 Further Case Studies 303
    14.3.1 From the General Resource Model to Petri Nets 303
    14.3.2 From Process Interaction Diagrams to Timed Petri Nets 304
  14.4 Conclusion 304

15 Implementation of Typed Attributed Graph Transformation by AGG 305
  15.1 Language Concepts of AGG 305
    15.1.1 Graphs 306
    15.1.2 Typing Facilities 306
    15.1.3 Node and Edge Attributes 307
    15.1.4 Rules and Matches 308
    15.1.5 Graph Transformations 310
    15.1.6 Graph Grammars 312
  15.2 Analysis Techniques Implemented in AGG 312
    15.2.1 Graph Constraints 312
    15.2.2 Critical Pair Analysis 313
    15.2.3 Graph Parsing 317
    15.2.4 Termination 318
  15.3 Tool Environment of AGG 318
    15.3.1 Visual Environment 320
    15.3.2 Graph Transformation Engine 321
    15.3.3 Tool Integration 322
  15.4 Conclusion 322
Appendices

A  A Short Introduction to Category Theory .......................... 329
   A.1 Categories .................................................. 329
   A.2 Construction of Categories, and Duality ...................... 330
   A.3 Monomorphisms, Epimorphisms, and Isomorphisms .......... 332
   A.4 Pushouts and Pullbacks ...................................... 334
   A.5 Binary Coproducts and Initial Objects ....................... 340
   A.6 Functors, Functor Categories, and Comma Categories ...... 344
   A.7 Isomorphism and Equivalence of Categories .................. 350

B  A Short Introduction to Signatures and Algebras ................. 353
   B.1 Algebraic Signatures ........................................ 353
   B.2 Algebras .................................................... 355
   B.3 Terms and Term Evaluation ................................... 357

C  Detailed Proofs .................................................. 359
   C.1 Completion of Proof of Fact 4.24 ............................ 359
   C.2 Proof of Lemma 6.25 ......................................... 361
   C.3 Completion of Proof of Theorem 11.3 ......................... 362
      C.3.1 Well-Definedness ....................................... 362
      C.3.2 Functors ................................................ 364
      C.3.3 Isomorphism ............................................ 365
   C.4 Proof of Lemma 11.17 ....................................... 366
      C.4.1 Well-Definedness ....................................... 366
      C.4.2 Pushout Property ....................................... 367
      C.4.3 Initial Pushout ......................................... 368
   C.5 Proof of Theorem 13.12 ..................................... 370
   C.6 Proof of Lemma 13.20 ....................................... 373

References ......................................................... 377

Index .............................................................. 385