# Contents

## PART I. FOUNDATIONS

### I.1 The Scope of Integer and Combinatorial Optimization

1. Introduction
2. Modeling with Binary Variables I: Knapsack, Assignment and Matching, Covering, Packing and Partitioning
3. Modeling with Binary Variables II: Facility Location, Fixed-Charge Network Flow, and Traveling Salesman
4. Modeling with Binary Variables III: Nonlinear Functions and Disjunctive Constraints
5. Choices in Model Formulation
6. Preprocessing
7. Notes
8. Exercises

### I.2 Linear Programming

1. Introduction
2. Duality
3. The Primal and Dual Simplex Algorithms
4. Subgradient Optimization
5. Notes

### I.3 Graphs and Networks

1. Introduction
2. The Minimum-Weight or Shortest-Path Problem
3. The Minimum-Weight Spanning Tree Problem
4. The Maximum-Flow and Minimum-Cut Problems
5. The Transportation Problem: A Primal-Dual Algorithm
6. A Primal Simplex Algorithm for Network Flow Problems
7. Notes

### I.4 Polyhedral Theory

1. Introduction and Elementary Linear Algebra
2. Definitions of Polyhedra and Dimension
3. Describing Polyhedra by Facets
4. Describing Polyhedra by Extreme Points and Extreme Rays
5. Polarity
6. Polyhedral Ties Between Linear and Integer Programs 104
7. Notes 109
8. Exercises 109

I.5 Computational Complexity 114
1. Introduction 114
2. Measuring Algorithm Efficiency and Problem Complexity 117
3. Some Problems Solvable in Polynomial Time 121
4. Remarks on 0-1 and Pure-Integer Programming 125
5. Nondeterministic Polynomial-Time Algorithms and \( \mathcal{NP} \) Problems 127
6. The Most Difficult \( \mathcal{NP} \) Problems: The Class \( \mathcal{NP}^c \) 131
7. Complexity and Polyhedra 139
8. Notes 142
9. Exercises 143

I.6 Polynomial-Time Algorithms for Linear Programming 146
1. Introduction 146
2. The Ellipsoid Algorithm 147
3. The Polynomial Equivalence of Separation and Optimization 161
4. A Projective Algorithm 164
5. A Strongly Polynomial Algorithm for Combinatorial Linear Programs 172
6. Notes 180

I.7 Integer Lattices 182
1. Introduction 182
2. The Euclidean Algorithm 184
3. Continued Fractions 187
4. Lattices and Hermite Normal Form 189
5. Reduced Bases 195
6. Notes 201
7. Exercises 202

PART II. GENERAL INTEGER PROGRAMMING 203

II.1 The Theory of Valid Inequalities 205
1. Introduction 205
2. Generating All Valid Inequalities 217
3. Gomory's Fractional Cuts and Rounding 227
4. Superadditive Functions and Valid Inequalities 229
5. A Polyhedral Description of Superadditive Valid Inequalities for Independence Systems 237
6. Valid Inequalities for Mixed-Integer Sets 242
7. Superadditivity for Mixed-Integer Sets 246
8. Notes 254
9. Exercises 256
**II.2 Strong Valid Inequalities and Facets for Structured Integer Programs**

1. Introduction 259  
2. Valid Inequalities for the 0-1 Knapsack Polytope 265  
3. Valid Inequalities for the Symmetric Traveling Salesman Polytope 270  
4. Valid Inequalities for Variable Upper-Bound Flow Models 281  
5. Notes 290  
6. Exercises 291

**II.3 Duality and Relaxation** 296

1. Introduction 296  
2. Duality and the Value Function 300  
3. Superadditive Duality 304  
4. The Maximum-Weight Path Formulation and Superadditive Duality 308  
5. Modular Arithmetic and the Group Problem 312  
6. Lagrangian Relaxation and Duality 323  
7. Benders' Reformulation 337  
8. Notes 341  
9. Exercises 343

**II.4 General Algorithms** 349

1. Introduction 349  
2. Branch-and-Bound Using Linear Programming Relaxations 355  
3. General Cutting-Plane Algorithms 367  
4. Notes 379  
5. Exercises 381

**II.5 Special-Purpose Algorithms** 383

1. Introduction 383  
2. A Cutting-Plane Algorithm Using Strong Valid Inequalities 386  
3. Primal and Dual Heuristic Algorithms 393  
4. Decomposition Algorithms 409  
5. Dynamic Programming 417  
6. Notes 424  
7. Exercises 427

**II.6 Applications of Special-Purpose Algorithms** 433

1. Knapsack and Group Problems 433  
2. 0-1 Integer Programming Problems 456  
3. The Symmetric Traveling Salesman Problem 469  
4. Fixed-Charge Network Flow Problems 495  
5. Applications of Basis Reduction 513  
6. Notes 520  
7. Exercises 526