Response Surface Methodology
Process and Product Optimization
Using Designed Experiments

Second Edition

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