CONTENTS

Preface		v
Contents		ix
Introduction		1
PART A: COMBINATORS AND TRUTH		11
I	Introducing operations	13
	1. The basic language	14
	2. Operations I: general facts	15
	3. Operations II: elementary recursion theory	18
	4A. The Church-Rosser theorem	22
	4B. Term models	26
	5. The graph model	28
	6. An effective version of the extensional model D_∞	34
	Appendix	39
II	Extending operations with reflective truth	43
	7. Extending combinatory algebras with truth	45
	8. The theory of operations and reflective truth: simple consequences	51
	9A. Type-free abstraction, predicates and classes	55
	9B. Operations on predicates and classes	59
	10A. The fixed point theorem for predicates	63
	10B. Applications to semantics and recursion theory	68
	11. Non-extensionality	73
	Appendix I: a property theoretic definition of the fixed point operator for predicates	76
	Appendix II: on the explicit abstraction theorem	77
	Appendix III: independence of truth predicates from the encoding of logical operators	80

PART B: TRUTH AND RECURSION THEORY 83

III Inductive models and definability theory 85
12. Inductive models and the induction theorem 86
13. The envelope of an inductive model 88
14. The uniform ordinal comparison theorem for inductive models 91
15. Applications of the uniform ordinal comparison theorem 97

IV Type-free abstraction with approximation operator 103
16. Approximating properties by classes 104
17. The approximation theorem for extensional operations and the fixed point theorem for monotone operations 109
18. Topology displayed: basic definitions 113
19. The representation theorem for explicitly CL-continuous operators 117
Appendix: alternative proofs 122

V Type-free abstraction, choice and sets 125
20. Choice principles and the distinction between operations and functions 126
21. Admissible hulls: elementary facts 131
22. A model of admissible set theory 137
23. The boundedness theorem 144

PART C: SELECTED TOPICS 149

VI Levels of implication and intensional logical equivalence 151
24. Myhill's levels of implication 152
25. Formal deducibility based on levels of implication and its proof-theoretic strength 158
26. Introducing an intensional equivalence relation 162
27. The infinitary reduction relation $\Rightarrow$ 165
28. The Church-Rosser theorem for $\Rightarrow$ 169
29. A model of type-free logic based on intensional equivalence 174

VII On the global structure of models for reflective truth 177
30. The lattice of fixed point models for the neutral minimal theory 179
31. The sublattice of intrinsic fixed point models and the cardinality theorem 186

32. Variations on the encoding technique: non-modularity and other oddities 192
33. A model for an impredicative extension of reflective truth 198
34. On Kripke's classification of self-referential sentences 203
35. On the consistency of coinduction principles 207
Appendix: a variant to the basic operator Γ and the restriction axiom 209

PART D: LEVELS OF TRUTH AND PROOF THEORY 213

VIII Levels of reflective truth 215
36. A language and axioms for reflective truth with levels 218
37. Simple consequences 220
38. Universes and the Weyl extended iteration principle 225
39. A recursion-theoretic model 230
40. Levels of truth and predicatively reducible subsystems of second-order arithmetic 238
41. Consistency of a reducibility principle for classes 244
42. Levels of truth and impredicative subsystems of second-order arithmetic 248
Appendix: on projectibility and stronger reflection 253

IX Levels of truth and predicative well-orderings 257
43. On well-orderings 258
44. Ramified hierarchies 261
45. Predicative well-orderings I 269
46. Predicative well-orderings II 277

X Reducing reflective truth with levels to finitely iterated reflective truth 285
47. A sequent calculus STLR for a theory of reflective truth with levels 286
48. Basic properties of STLR 291
49A. Elimination of the full level induction schema 293
49B. Elimination of unbounded level quantifiers 297
50. The infinitary sequent calculus IT_n^∞ of n-iterated reflective truth 303
51. Embedding STLR_n into IT_n^∞ 305

XI Proof-theoretic investigation of finitely iterated reflective truth 311
52. The ramified system RS_n 312
53. Cut elimination 316
54. Some derivable sequents of RS_n 320

55. Embedding IT_n^∞ into RS_n 324
56. The upper bound theorem for IT_n^∞ 327
57. Upper bound theorems for TLR and its subsystems 329
58. Conclusion: the conservation theorems 335
Appendix: primitive recursive cut elimination for RS_n 338

PART E: ALTERNATIVE VIEWS 349

XII Non-reductive systems for type-free abstraction and truth 351
59. The core system VF^- and transfinite induction 352
60. Supervaluation models of VF^- 357
61. An abstract sequent calculus and truth 358
62. Cut elimination and related properties 364
63. A provability interpretation and the upper bound theorem 369
64. Reconciling supervaluation models with provability interpretation 375

XIII The variety of non-reductive approaches 379
65. An inconsistency 380
66. On a truth theory of Friedman and Sheard 383
67. Fitch's models 386
68. Introducing semi-inductive definitions 390
69. Semi-inductive models for reflective truth 394

XIV Epilogue: applications and perspectives 401
70A. A logical theory of constructions: informal motivations 402
70B. A logical theory of constructions: basic syntax 403
71. Axioms for the computation relations 407
72. Extending the logical theory of constructions with higher reflection 411
73. Proof-theoretic reduction 416
74. Perspectives: related work in Artificial Intelligence and Theoretical Linguistics 419
75. Sense and denotation as algorithm and value: subsuming theories of reflective truth under abstract recursion theory 422

Bibliography 425

Index 441

List of Symbols 453