## CONTENTS

### Preface

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ix</td>
</tr>
</tbody>
</table>

1. How delays arise and what effects they have  
2. Functional differential equations  
3. Origin and effects of delays  
4. Example A: Neurophysiology of the retina  
5. Example B: Insect maturation times  
6. Example C: Maturation of blood cells  
7. Example D: Population models  
8. Example E: Incubation times in an epidemic model  
9. Example F: Neuron interaction model  
10. Example G: Chemostat models  
11. Example H: Discrete delay in a control system  
12. Example I: A simple system oscillating because of delay  
13. Example J: The damped linear oscillator with delayed restoring force  

2. Ordinary differential equations: the polynomial characteristic equation  

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
</tr>
</tbody>
</table>

1. Fixed points and linearisation  
2. Second-order linear systems  
3. nth-order linear equation  
4. n coupled linear equations of first order  
5. A simple example: the Monod chemostat model  
   Exercises  

3. Functional differential equations: the transcendental characteristic equation  

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
</tr>
</tbody>
</table>

1. Fixed points and linearisation  
2. The characteristic equation for discrete delays  
3. Geometry of simple cases  
4. The characteristic equation for one distributed delay  
5. Comparing the geometry of discrete and distributed delay  
   Exercises
## Contents

4 Hurwitz polynomials 60
   1 Hurwitz polynomials and associated functions 60
   2 The Routh table 65
   3 The Routh–Hurwitz determinants 67
   4 Stability boundaries 71
   5 Counting real and positive roots 75
   Exercises 80

5 First- and second-order systems with a discrete delay 82
   1 Stability boundaries 82
   2 The linear case 85
   3 The quadratic case 87
   4 Detailed treatment of the quadratic case 91
   5 More difficult problems 95
   Exercises 97

6 Higher-order systems, and systems with two delays 99
   1 Higher-order polynomials in characteristic equations 99
   2 Analytic $H(z)$ and $K(z)$: systems with two delays 104
   3 An interference effect of two delays 107
   4 Characteristic equations with $T$-dependent coefficients 109
   An extended exercise 112

7 Reducing a discrete delay problem to one with a polynomial characteristic equation 114
   1 Approximate delays and pseudo-delays 114
   2 Taylor expansion 117
   3 Padé approximants 119
   4 Product approximation 120
   5 The pseudo-delay method 120
   6 A problem with two delays 123
   7 Alternative version of the pseudo-delay method 126
   Exercises 127

8 Stability independent of delay 129
   1 Independent and commensurate delays 129
   2 Two delays: characteristic equation linear in $z$ 131
   3 The criteria of Hale, Infante and Tsen 133
   4 The Hermite matrix method 134
   5 The eliminant method 136
   6 Another treatment of the interference effect 142
   Exercises 146

9 Distributed delay 148
   1 Qualitative features of distributed delay 148
   2 Nonreducible distributions 151
Contents

3 The linear characteristic equation 158
4 Stability independent of mean delay 162
5 Distributed delay with a gap 163

Exercises 164

10 Reducible delays and linear subsystems 166
1 The linear chain 166
2 Linear subsystems interpreted as distributed delays 169
3 Approximating a distributed delay by a reducible delay 173

Appendices 175
1 The Laplace transform 175
2 Complex variable theory 180
3 Determinants 190
4 Pontriagin’s method 200
5 Neutral equations 203

Solutions to exercises 205
References 227
Index 233