Contents

Part I Construction Theory of Sample Functions of Homogeneous Denumerable Markov Processes .............................................. 1

Chapter I The First Construction Theorem ............................................. 3
§ 1.1 Introduction .................................................................................. 3
§ 1.2 Definition of transformation $g_n$ .................................................. 5
§ 1.3 Convergence of the sequence $X^{(n)}(\omega)$ $(n \geq 1)$ ...................... 6
§ 1.4 Further properties of $X^{(n)}(\omega)$ $(n \geq 1)$ ................................. 9
§ 1.5 The first construction theorem ...................................................... 14

Chapter II The Second Construction Theorem ...................................... 16
§ 2.1 Introduction .................................................................................. 16
§ 2.2 The mapping $T_{mn}$ ..................................................................... 16
§ 2.3 The mapping $W_n$ ....................................................................... 19
§ 2.4 Constructing auxiliary functions .................................................... 24
§ 2.5 The second construction theorem .................................................. 26
§ 2.6 Summary ...................................................................................... 26
§ 2.7 Two notes ..................................................................................... 27

Part II Theory of Minimal Nonnegative Solutions for Systems of Nonnegative Linear Equations .................................................. 29

Chapter III General Theory ................................................................. 31
§ 3.1 Introduction .................................................................................. 31
§ 3.2 Definition of a system of nonnegative linear equations and definition, existence and uniqueness of its minimal nonnegative solution .......................................................................................................................... 31
§ 3.3 Comparison theorem and linear combination theorem ...................... 34
§ 3.4 Localization theorem ..................................................................... 36
§ 3.5 Connecting property of the minimal nonnegative solution .............. 36
§ 3.6 Limit theorem ................................................................................ 38
§ 3.7 Matrix representation .................................................................... 39
§ 3.8 Dual theorem ................................................................................ 39
Chapter IV Calculation

§ 4.1 Some lemmas ........................................ 41
§ 4.2 Reduction of the problems ......................... 43
§ 4.3 Ordinary systems of strictly nonhomogeneous equations with dimension $n$ ........................................ 45

Chapter V Systems of 1-Bounded Equations ......................... 47

§ 5.1 Introduction ........................................ 47
§ 5.2 First-type leading-outside systems of equations ........ 48
§ 5.3 First-type consistent systems of equations ............ 49
§ 5.4 Tailed random systems of strictly nonhomogeneous equations ........................................ 51
§ 5.5 Regular systems of equations ......................... 52
§ 5.6 Pseudo-normal systems of equations .................. 54
§ 5.7 Pseudo-normal systems of equations of finite dimension 56
§ 5.8 Second-type regular systems of equations ............. 61

Part III Homogeneous Denumerable Markov Chains ............... 65

Chapter VI General Theory ..................................... 67

§ 6.1 Introduction ........................................ 67
§ 6.2 Transition probabilities .............................. 67
§ 6.3 Distribution and moments of the first passage time ...... 69
§ 6.4 Distribution and moments of the first passage time of a homogeneous finite Markov chain ..................... 74
§ 6.5 Distribution and moments of the times of passage ...... 77
§ 6.6 Criteria for recurrence ................................ 79
§ 6.7 Distribution and moments of additive functionals ...... 82
§ 6.8 Derived Markov chains and criteria for atomic almost closed sets ........................................ 86

Chapter VII Martin Exit Boundary Theory ......................... 93

§ 7.1 Introduction ........................................ 93
§ 7.2 Decomposition for Markov chains ...................... 93
§ 7.3 Limit behaviour of excessive functions ................ 96
§ 7.4 Green functions and Martin kernels .................... 97
§ 7.5 $h$-chains ........................................ 99
§ 7.6 Limit theorem for Martin kernels ...................... 105
§ 7.7 Martin boundaries ................................ 107
§ 7.8 Distribution of $x_\zeta$ ................................ 111
§ 7.9 Martin expressions of excessive functions ............. 112
| § 7.10   | Exit space                                                                 | 113 |
| § 7.11   | Uniqueness theorem                                                          | 114 |
| § 7.12   | Minimal excessive functions                                                 | 114 |
| § 7.13   | Terminal random variables                                                   | 115 |
| § 7.14   | Criteria for potentials and excessive functions, Riesz decomposition        | 116 |
| § 7.15   | Criteria for minimal harmonic functions, minimal potentials and minimal excessive functions | 118 |
| § 7.16   | Atomic exit spaces and nonatomic exit spaces                                 | 120 |
| § 7.17   | Blackwell decomposition of the state space                                  | 122 |

Chapter VIII Martin Entrance Boundary Theory .................................................. 124

| § 8.1    | Introduction                                                               | 124 |
| § 8.2    | The first group of lemmas                                                  | 125 |
| § 8.3    | Properties of finite excessive measures                                   | 127 |
| § 8.4    | The second group of lemmas                                                 | 128 |
| § 8.5    | Entrance boundary                                                         | 131 |
| § 8.6    | Entrance space and the expressions of excessive measures                  | 131 |

Part IV Homogeneous Denumerable Markov Processes ............................................ 133

Chapter IX Minimal Q-Processes ........................................................................... 135

| § 9.1    | Introduction                                                               | 135 |
| § 9.2    | Transition probabilities                                                  | 135 |
| § 9.3    | Distribution and moments of the first passage time                         | 136 |
| § 9.4    | Criterion for the positive recurrence                                      | 144 |
| § 9.5    | Distribution and moments of integral-type functionals                      | 146 |
| § 9.6    | Distribution and moments of integral-type functionals on pseudo-translatable sets | 159 |
| § 9.7    | Extensions of the results in § 9.3                                         | 166 |

Chapter X Q-Processes of Order One .................................................................... 168

| § 10.1   | Introduction                                                               | 168 |
| § 10.2   | Transition probabilities                                                  | 169 |
| § 10.3   | Distribution and moments of the first passage time                         | 175 |

Chapter XI Arbitrary Q-Processes ...................................................................... 186

| § 11.1   | Strengthening of the first construction theorem                            | 186 |
| § 11.2   | Transition probability                                                     | 192 |
| § 11.3   | Decomposition theorems for excessive measures and excessive functions      | 194 |
Part V  Construction Theory of Homogeneous Denumerable
Markov Processes ........................................... 201

Chapter XII Criteria for the Uniqueness of $Q$-Processes ................. 203
$\S\ 12.1$ Introduction ............................................ 203
$\S\ 12.2$ Lemmas .................................................. 205
$\S\ 12.3$ Proof of the main theorem ................................ 210
$\S\ 12.4$ The case of diagonal type ................................ 219
$\S\ 12.5$ The bounded case ........................................ 220
$\S\ 12.6$ The case when $E$ is finite ................................ 222
$\S\ 12.7$ The case of a branch $Q$-matrix ......................... 222
$\S\ 12.8$ Another criterion and the finite and nonconservative case .... 228
$\S\ 12.9$ Independence of the two conditions in Theorem 12.1.1 .... 230
$\S\ 12.10$ Probability interpretation of Condition (i) in
Theorem 12.1.1 .................................................. 232

Chapter XIII Construction of $Q$-Processes ........................................ 240
$\S\ 13.1$ Construction theorem .................................... 240
$\S\ 13.2$ Specifications of all the $Q$-processes .................... 242
$\S\ 13.3$ Expression of $\{Q, \Pi_{(0,\infty)\times E}\}$-processes .......... 243
$\S\ 13.4$ Discussion ................................................ 243

Chapter XIV Qualitative Theory ............................................. 246
$\S\ 14.1$ Introduction .............................................. 246
$\S\ 14.2$ Statement of results ..................................... 247
$\S\ 14.3$ Reduction of the construction problem of $B$-type $Q$-
processes, Doob processes ...................................... 253
$\S\ 14.4$ Reduction of the construction problem of $B \cap F$-type
$Q$-processes .................................................... 260
$\S\ 14.5$ Proofs of Theorems 14.2.1–14.2.3 .......................... 263
$\S\ 14.6$ Proof and examples of applications of Theorem 14.2.4 .... 263
$\S\ 14.7$ Proofs of Theorems 14.2.5–14.2.10 ........................ 275

Bibliography .................................................................. 279

Index .......................................................................... 281