

# Contents

*Preface*

ix

## **Chapter 1 Perspectives and Problems of Nonlinear System Theory**

I. A Nonlinear World	1
II. The Central Questions of System Theory	10
III. Classes of Nonlinear Processes	18
IV. Linear versus Nonlinear System Theory	19
Notes and References	23

## **Chapter 2 Mathematical Tools of Nonlinear System Theory**

I. Introduction	25
II. Algebraic Concepts	25
III. Some Ideas from Differential Geometry	36
IV. Concepts from Algebraic Geometry	44
V. Grassmann Manifolds and the Classification Problem	54
Problems and Exercises	62
Notes and References	66

## **Chapter 3 A Modern View of Linear System Theory**

I. Introduction	68
II. Mathematical Description of a Linear System	69
III. The Module Structure of $\Theta$ , $\Gamma$ , and $\Psi$	70
IV. Some System-Theoretic Consequences	75
V. Transfer Functions	78
VI. Realization of Transfer Functions	82
VII. The Construction of Canonical Realizations	85
VIII. Partial Realizations	92

IX. Pole-Shifting and Stability	94
X. Some Geometric Aspects of Linear Systems	95
XI. Feedback, the McMillan Degree, and Kronecker Indices	99
Problems and Exercises	102
Notes and References	106

## Chapter 4 Reachability and Controllability

I. Introduction	110
II. Basic Definitions and Problem Statement	112
III. Solution Manifolds for Dynamical Systems	117
IV. Some General Results	121
V. Linear-Analytic Systems	124
VI. The Role of the State Manifold $M$	126
VII. Bilinear Systems	127
VIII. Polynomial Systems	130
IX. Discrete-Time Systems	133
X. Input Construction	135
XI. Control Canonical Form and System Invariants	137
Problems and Exercises	141
Notes and References	146

## Chapter 5 Observability, Realization, and Estimation

I. Measurements and State Determination	150
II. Some Basic Definitions	152
III. Basic Observability Results	154
IV. Polynomial Systems	159
V. Realization Theory	162
VI. Nonlinear Filtering and Estimation	170
Problems and Exercises	177
Notes and References	186

## Chapter 6 Stability Theory: Singularities, Bifurcations, and Catastrophes

I. Concepts of Stability	190
II. Lyapunov Stability	192
III. The Center Manifold Theorem	200
IV. Families of Vector Fields and Normal Forms	203
V. Elementary Bifurcations	206
VI. Singularity Theory and the Elementary Catastrophes	212
VII. Applications of Catastrophe Theory	219
VIII. Chaos and Strange Attractors	226
IX. Control Systems and Feedback	232
Problems and Exercises	240
Notes and References	252