Contents

Introduction

List of Figures xiii

List of Tables xvii

List of Symbols and Notations xxi

I From Portfolio Optimization to Risk Parity 1

1 Modern Portfolio Theory 3

1.1 From optimized portfolios to the market portfolio 4

1.1.1 The efficient frontier 4

1.1.1.1 Introducing the quadratic utility function 6

1.1.1.2 Adding some constraints 9

1.1.1.3 Analytical solution 11

1.1.2 The tangency portfolio 12

1.1.3 Market equilibrium and CAPM 16

1.1.4 Portfolio optimization in the presence of a benchmark 19

1.1.5 The Black-Litterman model 22

1.1.5.1 Computing the implied risk premia 23

1.1.5.2 The optimization problem 24

1.1.5.3 Numerical implementation of the model 25

1.2 Practice of portfolio optimization 27

1.2.1 Estimation of the covariance matrix 27

1.2.1.1 Empirical covariance matrix estimator 27

1.2.1.2 Hayashi-Yoshida estimator 29

1.2.1.3 GARCH approach 32

1.2.1.4 Factor models 35

1.2.2 Designing expected returns 40

1.2.3 Regularization of optimized portfolios 44

1.2.3.1 Stability issues 45

1.2.3.2 Resampling techniques 45

1.2.3.3 Denoising the covariance matrix 47

1.2.3.4 Shrinkage methods 49

1.2.4 Introducing constraints 53
1.2.4.1 Why regularization techniques are not sufficient 54
1.2.4.2 How to specify the constraints 57
1.2.4.3 Shrinkage interpretation of the constrained solution 65

2 Risk Budgeting Approach 71

2.1 Risk allocation principle 72
2.1.1 Properties of a risk measure 72
2.1.1.1 Coherency and convexity of risk measures 72
2.1.1.2 Euler allocation principle 77
2.1.2 Risk contribution of portfolio assets 79
2.1.2.1 Computing the risk contributions 79
2.1.2.2 Interpretation of risk contributions 82
2.1.3 Application to non-normal risk measures 84
2.1.3.1 Non-normal value-at-risk and expected shortfall 84
2.1.3.2 Historical value-at-risk 92

2.2 Analysis of risk budgeting portfolios 97
2.2.1 Definition of a risk budgeting portfolio 98
2.2.1.1 The right specification of the RB portfolio 99
2.2.1.2 Solving the non-linear system of risk budgeting constraints 102
2.2.2 Some properties of the RB portfolio 102
2.2.2.1 Particular solutions with the volatility risk measure 102
2.2.2.2 Existence and uniqueness of the RB portfolio 108
2.2.3 Optimality of the risk budgeting portfolio 113
2.2.4 Stability of the risk budgeting approach 116

2.3 Special case: the ERC portfolio 119
2.3.1 The two-asset case \((n - 2)\) 119
2.3.2 The general case \((n > 2)\) 121
2.3.3 Optimality of the ERC portfolio 123
2.3.4 Back to the notion of diversification 125
2.3.4.1 Diversification index 125
2.3.4.2 Concentration indices 126
2.3.4.3 Difficulty of reconciling the different diversification concepts 128

2.4 Risk budgeting versus weight budgeting 130
2.4.1 Comparing weight budgeting and risk budgeting portfolios 130
2.4.2 New construction of the minimum variance portfolio 131

2.5 Using risk factors instead of assets 135
2.5.1 Pitfalls of the risk budgeting approach based on assets 135
2.5.1.1 Duplication invariance property 135
2.5.1.2 Polico invariance property 137
2.5.1.3 Impact of the reparametrization on the asset universe 138
2.5.2 Risk decomposition with respect to the risk factors 141
2.5.3 Some illustrations 144
2.5.3.1 Matching the risk budgets 144
2.5.3.2 Minimizing the risk concentration between the risk factors 145
2.5.3.3 Solving the duplication and polico invariance properties 146

II Applications of the Risk Parity Approach 149

3 Risk-Based Indexation 151
3.1 Capitalization-weighted indexation 152
3.1.1 Theory support 152
3.1.2 Constructing and replicating an equity index 153
3.1.3 Pros and cons of CW indices 154
3.2 Alternative-weighted indexation 157
3.2.1 Desirable properties of AW indices 159
3.2.2 Fundamental indexation 160
3.2.3 Risk-based indexation 162
3.2.3.1 The equally weighted portfolio 163
3.2.3.2 The minimum variance portfolio 164
3.2.3.3 The most diversified portfolio 168
3.2.3.4 The ERC portfolio 172
3.2.3.5 Comparison of the risk-based allocation approaches 173
3.3 Some illustrations 181
3.3.1 Simulation of risk-based indices 181
3.3.2 Practical issues of risk-based indexation 183
3.3.3 Findings of other empirical works 187
3.3.3.1 What is the best alternative-weighted indexation? 187
3.3.3.2 Style analysis of alternative-weighted indexation 189

4 Application to Bond Portfolios 191
4.1 Some issues in bond management 191
4.1.1 Debt-weighted indexation 191
4.1.2 Yield versus risk 193
4.2 Bond portfolio management 194
4.2.1 Term structure of interest rates 194
4.2.2 Pricing of bonds 197
4.2.2.1 Without default risk 197
4.2.2.2 With default risk 200
4.2.3 Risk management of bond portfolios 203
  4.2.3.1 Using the yield curve as risk factors 204
  4.2.3.2 Taking into account the default risk 209
4.3 Some illustrations 215
  4.3.1 Managing risk factors of the yield curve 216
  4.3.2 Managing sovereign credit risk 220
    4.3.2.1 Measuring the credit risk of sovereign bond portfolios 222
    4.3.2.2 Comparing debt-weighted, gdp-weighted and risk-based indexations 231

5 Risk Parity Applied to Alternative Investments 243
  5.1 Case of commodities 244
    5.1.1 Why investing in commodities is different 244
      5.1.1.1 Commodity futures markets 244
      5.1.1.2 How to define the commodity risk premium 246
    5.1.2 Designing an exposure to the commodity asset class 247
      5.1.2.1 Diversification return 247
      5.1.2.2 Comparing EW and ERC portfolios 251
  5.2 Hedge fund strategies 254
    5.2.1 Position sizing 254
    5.2.2 Portfolio allocation of hedge funds 257
      5.2.2.1 Choosing the risk measure 258
      5.2.2.2 Comparing ERC allocations 258
      5.2.2.3 Budgeting the risk factors 262
      5.2.2.4 Limiting the turnover 265

6 Portfolio Allocation with Multi-Asset Classes 269
  6.1 Construction of diversified funds 270
    6.1.1 Stock/bond asset mix policy 270
    6.1.2 Growth assets versus hedging assets 273
      6.1.2.1 Are bonds growth assets or hedging assets 273
      6.1.2.2 Analytics of these results 277
    6.1.3 Risk-balanced allocation 278
    6.1.4 Pros and cons of risk parity funds 280
  6.2 Long-term investment policy 284
    6.2.1 Capturing the risk premia 285
    6.2.2 Strategic asset allocation 286
      6.2.2.1 Allocation between asset classes 286
      6.2.2.2 Asset classes or risk factor classes 288
      6.2.2.3 Allocation within an asset class 291
    6.2.3 Risk budgeting with liability constraints 294
  6.3 Absolute return and active risk parity 294
A Technical Appendix

A.1 Optimization problems
  A.1.1 Quadratic programming problem
  A.1.2 Non-linear unconstrained optimization
  A.1.3 Sequential quadratic programming algorithm
  A.1.4 Numerical solutions of the RB problem

A.2 Copula functions
  A.2.1 Definition and main properties
  A.2.2 Parametric functions
  A.2.3 Simulation of copula models
    A.2.3.1 Distribution approach
    A.2.3.2 Simulation based on conditional copula functions

A.3 Dynamic portfolio optimization
  A.3.1 Stochastic optimal control
    A.3.1.1 Bellman approach
    A.3.1.2 Martingale approach
  A.3.2 Portfolio optimization in continuous-time
  A.3.3 Some extensions of the Merton model
    A.3.3.1 Lifestyle funds
    A.3.3.2 Lifecycle funds
    A.3.3.3 Liability driven investment

B Tutorial Exercises

B.1 Exercises related to modern portfolio theory
  B.1.1 Markowitz optimized portfolios
  B.1.2 Variations on the efficient frontier
  B.1.3 Sharpe ratio
  B.1.4 Beta coefficient
  B.1.5 Tangency portfolio
  B.1.6 Information ratio
  B.1.7 Building a tilted portfolio
  B.1.8 Implied risk premium
  B.1.9 Black-Litterman model
  B.1.10 Portfolio optimization with transaction costs
  B.1.11 Impact of constraints on the CAPM theory
  B.1.12 Generalization of the Jagannathan-Ma shrinkage approach

B.2 Exercises related to the risk budgeting approach
  B.2.1 Risk measures
  B.2.2 Weight concentration of a portfolio
B.2.3 ERC portfolio 353
B.2.4 Computing the Cornish-Fisher value-at-risk 354
B.2.5 Risk budgeting when risk budgets are not strictly positive 355
B.2.6 Risk parity and factor models 356
B.2.7 Risk allocation with the expected shortfall risk measure 358
B.2.8 ERC optimization problem 359
B.2.9 Risk parity portfolios with skewness and kurtosis 360
B.3 Exercises related to risk parity applications 362
B.3.1 Computation of heuristic portfolios 362
B.3.2 Equally weighted portfolio 362
B.3.3 Minimum variance portfolio 363
B.3.4 Most diversified portfolio 365
B.3.5 Risk allocation with yield curve factors 366
B.3.6 Credit risk analysis of sovereign bond portfolios 368
B.3.7 Risk contributions of long-short portfolios 370
B.3.8 Risk parity funds 371
B.3.9 The Frazzini-Pedersen model 372
B.3.10 Dynamic risk budgeting portfolios 374

Bibliography 377

Subject Index 399

Author Index 405