PRESSURE TRANSIENT FORMATION AND WELL TESTING

Convolution, Deconvolution and Nonlinear Estimation

By

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