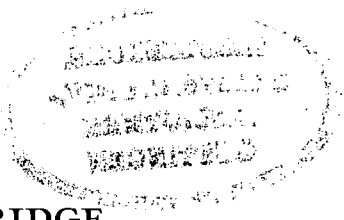


Orthogonal Polynomials and Continued Fractions

From Euler's Point of View

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