Exact and Approximate Controllability for Distributed Parameter Systems

A Numerical Approach

ROLAND GLOWINSKI
University of Houston

JACQUES-LOUIS LIONS
College de France, Paris

JIWEN HE
University of Houston

CAMBRIDGE
UNIVERSITY PRESS

Contents

Preface *page* xi

Introduction 1
I.1 What it is all about? 1
I.2 Motivation 2
I.3 Topologies and numerical methods 3
I.4 Choice of the control 4
I.5 Relaxation of the controllability notion 4
I.6 Various remarks 5

Part I Diffusion Models

1 Distributed and pointwise control for linear diffusion equations 9
1.1 First example 9
1.2 Approximate controllability 12
1.3 Formulation of the approximate controllability problem 14
1.4 Dual problem 15
1.5 Direct solution to the dual problem 17
1.6 Penalty arguments 19
1.7 L^∞ cost functions and bang-bang controls 22
1.8 Numerical methods 28
1.9 Relaxation of controllability 57
1.10 Pointwise control 62
1.11 Further remarks (I): Additional constraints on the state function 96
1.12 Further remarks (II): A bisection based memory saving method for the solution of time dependent control problems by adjoint equation based methodologies 112
1.13 Further remarks (III): A brief introduction to Riccati equations based control methods 117

2 Boundary control 124
2.1 Dirichlet control (I): Formulation of the control problem 124
2.2 Dirichlet control (II): Optimality conditions and dual formulations 126
2.3 Dirichlet control (III): Iterative solution of the control problems 128
2.4 Dirichlet control (IV): Approximation of the control problems 133
2.5 Dirichlet control (V): Iterative solution of the fully discrete dual problem (2.124) 143
2.6 Dirichlet control (VI): Numerical experiments 146
2.7 Neumann control (I): Formulation of the control problems and synopsis 155
2.8 Neumann control (II): Optimality conditions and dual formulations 163
2.9 Neumann control (III): Conjugate gradient solution of the dual problem (2.192) 176
2.10 Neumann control (IV): Iterative solution of the dual problem (2.208), (2.209) 178
2.11 Neumann control of unstable parabolic systems: a numerical approach 178
2.12 Closed-loop Neumann control of unstable parabolic systems via the Riccati equation approach 223

3 Control of the Stokes system 231
3.1 Generalities. Synopsis 231
3.2 Formulation of the Stokes system. A fundamental controllability result 231
3.3 Two approximate controllability problems 234
3.4 Optimality conditions and dual problems 234
3.5 Iterative solution of the control problem (3.19) 236
3.6 Time discretization of the control problem (3.19) 238
3.7 Numerical experiments 239

4 Control of nonlinear diffusion systems 243
4.1 Generalities. Synopsis 243
4.2 Example of a noncontrollable nonlinear system 243
4.3 Pointwise control of the viscous Burgers equation 245
4.4 On the controllability and the stabilization of the Kuramoto-Sivashinsky equation in one space dimension 259

5 Dynamic programming for linear diffusion equations 277
5.1 Introduction. Synopsis 277
5.2 Derivation of the Hamilton–Jacobi–Bellman equation 278
5.3 Some remarks 279

Part II Wave Models

6 Wave equations 283

6.1 Wave equations: Dirichlet boundary control 283
6.2 Approximate controllability 285
6.3 Formulation of the approximate controllability problem 286
6.4 Dual problems 287
6.5 Direct solution of the dual problem 288
6.6 Exact controllability and new functional spaces 289
6.7 On the structure of space E 291
6.8 Numerical methods for the Dirichlet boundary controllability of the wave equation 291
6.9 Experimental validation of the filtering procedure of Section 6.8.7 via the solution of the test problem of Section 6.8.5 315
6.10 Some references on alternative approximation methods 319
6.11 Other boundary controls 320
6.12 Distributed controls for wave equations 328
6.13 Dynamic programming 329

7 On the application of controllability methods to the solution of the Helmholtz equation at large wave numbers 332

7.1 Introduction 332
7.2 The Helmholtz equation and its equivalent wave problem 332
7.3 Exact controllability methods for the calculation of time-periodic solutions to the wave equation 334
7.4 Least-squares formulation of the problem (7.8)–(7.11) 334
7.5 Calculation of J' 336
7.6 Conjugate gradient solution of the least-squares problem (7.14) 337
7.7 A finite element–finite difference implementation 340
7.8 Numerical experiments 341
7.9 Further comments. Description of a mixed formulation based variant of the controllability method 349
7.10 A final comment 355

8 Other wave and vibration problems. Coupled systems 356

8.1 Generalities and further references 356
8.2 Coupled Systems (I): a problem from thermo-elasticity 359
8.3 Coupled systems (II): Other systems 367

Part III Flow Control

9 Optimal control of systems modelled by the Navier–Stokes equations: Application to drag reduction 371
9.1 Introduction. Synopsis 371
9.2 Formulation of the control problem 373
9.3 Time discretization of the control problem 377
9.4 Full discretization of the control problem 379
9.5 Gradient calculation 384
9.6 A BFGS algorithm for solving the discrete control problem 388
9.7 Validation of the flow simulator 389
9.8 Active control by rotation 394
9.9 Active control by blowing and suction 408
9.10 Further comments on flow control and conclusion 419

Epilogue 426

Further Acknowledgements 429

References 430

Index of names 450

Index of subjects 454