Enumeration of Finite Groups
## Preface

1. Introduction

## ELEMENTARY RESULTS

2. Some basic observations

## GROUPS OF PRIME POWER ORDER

3. Preliminaries
   - 3.1 Tensor products and exterior squares of abelian groups
   - 3.2 Commutators and nilpotent groups
   - 3.3 The Frattini subgroup
   - 3.4 Linear algebra

4. Enumerating $p$-groups: a lower bound
   - 4.1 Relatively free groups
   - 4.2 Proof of the lower bound

5. Enumerating $p$-groups: upper bounds
   - 5.1 An elementary upper bound
   - 5.2 An overview of the Sims approach
   - 5.3 'Linearising' the problem
   - 5.4 A small set of relations
   - 5.5 Proof of the upper bound
Contents

13.4 The subgroup $B$ 119
13.5 Structure of $G$ determined by $B$ 125

14 Conjugacy classes of maximal soluble subgroups of the general linear groups 127

15 Pyber’s theorem: the soluble case 132
15.1 Extensions and soluble subgroups 133
15.2 Pyber’s theorem 135

16 Pyber’s theorem: the general case 140
16.1 Three theorems on group generation 140
16.2 Universal central extensions and covering groups 146
16.3 The generalised Fitting subgroup 150
16.4 The general case of Pyber’s theorem 154

IV OTHER TOPICS 161

17 Enumeration within varieties of abelian groups 163
17.1 Varieties of abelian groups 164
17.2 Enumerating partitions 167
17.3 Further results on abelian groups 173

18 Enumeration within small varieties of A-groups 174
18.1 A minimal variety of A-groups 175
18.2 The join of minimal varieties 184

19 Enumeration within small varieties of $p$-groups 187
19.1 Enumerating two small varieties 189
19.2 The ratio of two enumeration functions 191

20 Miscellanea 195
20.1 Enumerating $d$-generator groups 195
20.2 Groups with few non-abelian composition factors 206
20.3 Enumerating graded Lie rings 211
20.4 Groups of nilpotency class 3 216

21 Survey of other results 222
21.1 Graham Higman’s PORC conjecture 222
21.2 Isoclinism classes of $p$-groups 224
21.3 Groups of square-free order 227
21.4 Groups of cube-free order 233
21.5 Groups of arithmetically small orders 236
21.6 Surjectivity of the enumeration function 238
21.7 Densities of certain sets of group orders 246
21.8 Enumerating perfect groups 256

22 Some open problems 259

Appendix A: Maximising two functions 269

References 275

Index 280