## Contents

### Preface

1 Preliminaries

1.1 Differential calculus 1
1.2 Function spaces 4
1.3 Nemitski operators 5
1.4 Elliptic equations 7

Part I Topological methods

2 A primer on bifurcation theory

2.1 Bifurcation: definition and necessary conditions 15
2.2 The Lyapunov–Schmidt reduction 18
2.3 Bifurcation from the simple eigenvalue 19

3 Topological degree, I

3.1 Brouwer degree and its properties 26
3.2 Application: the Brouwer fixed point theorem 30
3.3 An analytic definition of the degree 31
3.4 The Leray–Schauder degree 38
3.5 The Schauder fixed point theorem 43
3.6 Some applications of the Leray–Schauder degree to elliptic equations 44
3.7 The Krasnoselski bifurcation theorem 52
3.8 Exercises 54

4 Topological degree, II: global properties

4.1 Improving the homotopy invariance 55
4.2 An application to a boundary value problem with sub- and super-solutions 57
9.2 Lusternik–Schnirelman theorems 147
9.3 Exercises 155

10 Critical points of even functionals on symmetric manifolds 157
10.1 The Krasnoselski genus 157
10.2 Existence of critical points 160
10.3 Multiple critical points of even unbounded functionals 164
10.4 Applications to Dirichlet boundary value problems 170
10.5 Exercises 176

11 Further results on elliptic Dirichlet problems 177
11.1 Radial solutions of semilinear elliptic equation on $\mathbb{R}^n$ 177
11.2 Boundary value problems with critical exponent 180
11.3 Discontinuous nonlinearities 188
11.4 Problems with concave-convex nonlinearities 198
11.5 Exercises 203

12 Morse theory 204
12.1 A short review of basic facts in algebraic topology 204
12.2 The Morse inequalities 212
12.3 An application: bifurcation for variational operators 224
12.4 Morse index of mountain pass critical points 229
12.5 Exercises 235

Part IV Appendices 233

Appendix 1 Qualitative results 241
Appendix 2 The concentration compactness principle 252
Appendix 3 Bifurcation for problems on $\mathbb{R}^n$ 262
Appendix 4 Vortex rings in an ideal fluid 274
Appendix 5 Perturbation methods 286
Appendix 6 Some problems arising in differential geometry 302

References 309

Index 315