Topics in Differential Topology

Amiya Mukherjee
Indian Statistical Institute
Kolkata
4.2. Geodesics on a manifold 127
4.3. Riemannian connection and geodesics 132
4.4. Exponential maps 139
4.5. Hopf-Rinow theorem 144
4.6. Totally geodesic submanifolds 147

Chapter 5. VECTOR BUNDLES ON MANIFOLDS 150
5.1. Vector bundles 150
5.2. Construction of vector bundles 161
5.3. Homotopy property of vector bundles 164
5.4. Subbundle and quotient bundle 167
5.5. Orientation 171
5.6. Reduction of structure group of a vector bundle 179
5.7. Homology characterisation of orientation 181

Chapter 6. TRANSVERSALITY 186
6.1. $\epsilon$-neighbourhood of submanifold of Euclidean space 186
6.2. Transversality 191
6.3. Compact one-manifolds and Brouwer’s theorem 198
6.4. Boundary and pre-image orientations 202
6.5. Intersection numbers, and degrees of maps 205
6.6. Hopf degree theorem 213

Chapter 7. TUBULAR NEIGHBOURHOODS 222
7.1. Tubular neighbourhood theorems 222
7.2. Collar neighbourhoods 225
7.3. Isotopy extension theorem 231
7.4. Uniqueness of tubular neighbourhoods 237
7.5. Manifolds with corners and straightening them 243
7.6. Construction of manifolds by gluing process 246

Chapter 8. SPACES OF SMOOTH MAPS 252
8.1. Spaces of jets 252
8.2. Weak and strong topologies 262
8.3. Continuity of maps between spaces of smooth maps 270
8.4. Spaces of immersions and embeddings 275
8.5. Baire property of the space of smooth maps 280
8.6. Smooth structures on jet spaces 282
8.7. Thom’s transversality theorem 287
8.8. Multi-jet transversality 293
8.9. Whitney’s immersion and embedding theorems 297
Chapter 9. MORSE THEORY
  9.1. Morse functions 299
  9.2. Critical levels and attaching handles 305
  9.3. Morse inequalities 321
  9.4. Perfect Morse functions 329
  9.5. Triangulations of manifolds 331

Chapter 10. THEORY OF HANDLE PRESENTATIONS 337
  10.1. Existence of handle presentation 338
  10.2. Duality theorems 344
  10.3. Normalisation of presentation 353
  10.4. Cancellation of handles 355
  10.5. Classification of closed surfaces 361
  10.6. Removal of intersection points 364
  10.7. Addition of handles 374
  10.8. Simplification of handle presentations 378
  10.9. $h$-cobordism and generalised Poincaré conjecture 382

Chapter 11. HOMOTOPIY CLASSIFICATION OF
  REGULAR SECTIONS 386
  11.1. Introduction 386
  11.2. Proof of Gromov's theorem 392
  11.3. Gromov's theorem for a disk $D^n$ 403
  11.4. First fibration theorem 404
  11.5. Second fibration theorem 406
  11.6. Third fibration theorem 407
  11.7. Gromov's theorem for closed manifolds 411
  11.8. Completion of the proof of Theorem 11.7.2 422
  11.9. Applications 427

Bibliography 432

Index 436