### Contents

_Preface_  
Preface  
page xiii

_Acknowledgments_  
xxi

**PART I: THE LAGRANGIAN FORMULATION**  
1  
**Lagrangian kinematics**  
1.1 Conservation of particle identity  
1.2 Streaklines, streamlines and steady flow  
1.3 Local kinematics  
5

2  
**Lagrangian statistics**  
2.1 Single-particle, single-time statistics  
2.2 Single-particle, two-time statistics  
2.3 Two-particle, two-time statistics  
2.4 The Eulerian–Lagrangian problem: path integrals  
16

3  
**Lagrangian dynamics**  
3.1 Conservation of mass  
3.2 Conservation of momentum  
3.3 Conservation of energy  
3.4 Variational principle  
3.5 Bernoulli’s theorem  
3.6 Kelvin’s theorem  
3.7 Cauchy–Weber integrals  
3.7.1 First integrals  
3.7.2 Matrix formulation  
3.7.3 Cauchy–Weber integrals and Clebsch potentials  
25

vii
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.8</td>
<td>Potential flow and a Riemannian metric</td>
<td>41</td>
</tr>
<tr>
<td>3.9</td>
<td>Boundary conditions</td>
<td>43</td>
</tr>
<tr>
<td>3.9.1</td>
<td>Rigid boundaries</td>
<td>43</td>
</tr>
<tr>
<td>3.9.2</td>
<td>Comoving boundaries</td>
<td>43</td>
</tr>
<tr>
<td>3.9.3</td>
<td>Comoving boundary conditions</td>
<td>44</td>
</tr>
<tr>
<td>3.9.4</td>
<td>Adjacent Lagrangian coordinates</td>
<td>48</td>
</tr>
<tr>
<td>3.10</td>
<td>Local dynamics</td>
<td>48</td>
</tr>
<tr>
<td>3.11</td>
<td>Relabeling symmetry</td>
<td>50</td>
</tr>
<tr>
<td>3.12</td>
<td>Historical note</td>
<td>55</td>
</tr>
<tr>
<td>4</td>
<td>Coordinates</td>
<td>56</td>
</tr>
<tr>
<td>4.1</td>
<td>Independent variables</td>
<td>56</td>
</tr>
<tr>
<td>4.2</td>
<td>Dependent space variables</td>
<td>57</td>
</tr>
<tr>
<td>4.3</td>
<td>Rotational symmetry</td>
<td>59</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Globally uniform rotations</td>
<td>59</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Time-varying rotations</td>
<td>59</td>
</tr>
<tr>
<td>5</td>
<td>Real fluids</td>
<td>62</td>
</tr>
<tr>
<td>5.1</td>
<td>Viscous stresses and heat conduction</td>
<td>62</td>
</tr>
<tr>
<td>5.2</td>
<td>Navier–Stokes equations for incompressible flow</td>
<td>62</td>
</tr>
<tr>
<td>5.3</td>
<td>Matrix formulation for viscous incompressible flow</td>
<td>64</td>
</tr>
<tr>
<td>5.4</td>
<td>Boundary conditions</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>Some analytical Lagrangian solutions</td>
<td>71</td>
</tr>
<tr>
<td>6.1</td>
<td>Flow around a cylinder</td>
<td>71</td>
</tr>
<tr>
<td>6.2</td>
<td>Gerstner’s trochoidal wave</td>
<td>72</td>
</tr>
<tr>
<td>6.3</td>
<td>One-dimensional gas dynamics</td>
<td>76</td>
</tr>
<tr>
<td>6.3.1</td>
<td>One-dimensional traveling waves</td>
<td>76</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Riemann invariants</td>
<td>77</td>
</tr>
<tr>
<td>6.3.3</td>
<td>Arbitrary one-dimensional flow</td>
<td>77</td>
</tr>
<tr>
<td>6.4</td>
<td>Plane Poiseuille flow</td>
<td>78</td>
</tr>
<tr>
<td>7</td>
<td>Sound waves, shear instabilities, Rossby waves and Ptolemaic vortices</td>
<td>79</td>
</tr>
<tr>
<td>7.1</td>
<td>Sound waves</td>
<td>79</td>
</tr>
<tr>
<td>7.2</td>
<td>Hydrodynamic stability</td>
<td>80</td>
</tr>
<tr>
<td>7.3</td>
<td>Rossby waves</td>
<td>82</td>
</tr>
<tr>
<td>7.4</td>
<td>Hamiltonian dynamics of Rossby waves</td>
<td>87</td>
</tr>
</tbody>
</table>

PART II: LAGRANGIAN FLOWS
8 Viscous incompressible flow
8.1 Simple shear flow
8.2 The suddenly accelerated plane wall: Stokes’ first problem
8.3 Flow near an oscillating flat plate: Stokes’ second problem
8.4 The boundary layer along a flat plate

9 General solvability
9.1 Kinematics
9.2 Incompressible dynamics (1)
9.3 Incompressible dynamics (2)
9.4 Incompressible dynamics (3)
9.5 Compressible dynamics
9.6 Labeling singularities
9.7 Phenomenology
9.8 Viscous incompressible flow
  9.8.1 Equations of motion
  9.8.2 Picard iteration
  9.8.3 A priori bounds
  9.8.4 The viscous operator
  9.8.5 The elliptic operator

PART III: DIFFUSION

10 Absolute dispersion
10.1 Displacement: first and second moments
10.2 Displacement pdf
10.3 Forward closure, boundary conditions
10.4 Backward closure, scalar concentrations
10.5 Reversibility for incompressible flow; the Markov property, Corrsin’s hypotheses
10.6 Scalar concentrations in compressible flow; floats, surface drifters and balloons
10.7 Corrections
10.8 Random flight models and plankton dynamics
10.9 Annual plankton patchiness
Contents

11 Relative dispersion 146
11.1 Joint displacement of a pair of particles 146
11.2 Separation of a pair of particles 150
11.3 Richardson's self-similar asymptotic solution 153
11.4 Lundgren's log normal solution 155
11.5 Observations of dispersion 158
11.6 Kinetic energy subranges 162
11.7 Kinetic energy spectra and structure functions 167
11.8 Kinetic energy spectra and longitudinal diffusivities 170

12 Convective subranges of the scalar variance spectrum 177
12.1 Scalar covariance 177
12.2 Reversibility 179
12.3 Power spectra 179
12.4 Enstrophy inertia convective subrange 181
12.5 Energy inertia convective subrange 182
12.6 Viscous convective subrange 185
12.7 Transition 186
12.8 Relative dispersion and plankton patchiness 188

13 Diffusion 191
13.1 Scalar diffusion: An approximate general solution 191
13.2 Variance spectrum 194
13.3 Enstrophy inertia diffusive subrange 196
13.4 Energy inertia diffusive subrange 198
13.5 Viscous diffusive subrange 200

PART IV: LAGRANGIAN DATA 207

14 Observing systems 211
14.1 The laboratory 211
14.2 The atmosphere 212
14.3 The ocean surface 212
14.4 The deep ocean 214

15 Data analysis: the single particle 216
15.1 Time series analysis: the single particle 216
15.1.1 Polarization of Lagrangian velocities 216
15.1.2 Diffusivities from floats 221
15.2 Assimilation: the single particle  
  15.2.1 Lagrangian measurement functionals  
  15.2.2 Lagrangian assimilation: first steps  

16 Data analysis: particle clusters  
  16.1 Time series analysis: the particle pair  
  16.2 Assimilation: particle clusters  
    16.2.1 Eulerian kinematical analysis  
    16.2.2 Lagrangian dynamical analysis:  
      shallow-water theory  
    16.2.3 Lagrangian dynamical analysis: Boussinesq  
      theory  
    16.2.4 Least-squares estimator  

References  

Subject Index  

Author Index