The Geometry of an Art

The History of the Mathematical Theory of Perspective from Alberti to Monge
Contents

Introduction

Key Issues
  Questions Concerning the History of Geometrical Perspective
  Questions Concerning Textbooks on Perspective
The Word ‘Perspective’
Other Publications
The Period and Regions Examined
The Sources and How They Are Used
Contexts and Restrictions
Conclusions

Acknowledgements

Colleagues, Students, and Friends
Institutions
Sources of Funding
Libraries

Notes to the Reader

Drawings and Notation
  Concepts Related to the Eye Point and the Picture Plane
  Concepts Related to Images of Points, Lines, and Planes
  Orthogonals, Transversals, and Verticals
  Rabatment
Mathematical Terminology, Results, and Techniques
  Lines and Line Segments
  Results from the Theory of Proportion
  Mathematical Techniques
  The Placement of the Mathematical Explanations
Bibliographies
  Two Bibliographies
  References, Orthography, and Ordering of Letters
Biographies
  Dates for the Protagonists
My Text
  Quotations and Paraphrases
  Use of My Earlier Publications
Chapter I. The Birth of Perspective

1.1. The First Written Account of Geometrical Perspective 1
1.2. The Origin of Perspective 2
1.3. Four Stimuli 3
   Painting a View 3
   Representation of Special Lines 4
   A Search for Mathematical Rules 10
   Inspiration from Optics 10
1.4. Brunelleschi 11
   Four Possible Techniques 11
   Brunelleschi's Conception of Perspective 13
   No Conclusion 13
   Brunelleschi's Success 14
1.5. Perspective Before the Renaissance? 15

Chapter II. Alberti and Piero della Francesca

II.1. The Two Earliest Authors 17
II.2. Alberti and His Work 17
   Alberti's Views on the Art of Painting 18
II.3. Alberti's Model 19
   Alberti's Two Methods of Producing a Perspective Image 21
II.4. Alberti's Construction 22
   The Representation of Orthogonals 22
   An Open Window 23
   A Scaled Unit 24
   Placement of the Centric Point 25
   The Images of the Transversals 25
   Choice of Parameters 28
   Alberti's Use of a Perspective Grid 28
II.5. Alberti's Theoretical Reflections and His Diagonal Rule 29
II.6. The Third Dimension in Alberti's Construction 33
II.7. Alberti's Construction in History 34
II.8. Piero della Francesca and His Work 34
   De Prospectiva Pingendi 36
II.9. The Theoretical Foundation of De Prospectiva 37
   The Angle Axiom 37
   Foreshortening of Orthogonals and Line Segments Parallel to \( \pi \) 38
   Piero on Visual Distortion 40
II.10. Piero and Alberti's Construction 40
   Piero's Rabatment 40
   Piero on the Correctness of the Construction 42
   Filarete and Francesco di Giorgio 43
II.11. Piero's Diagonal Construction 44
II.12. Piero's Distance Point Construction 46
   The Origin of Distance Point Constructions 46
   Piero on the Correctness of His Distance Point Construction 48
II.13. The Division Theorem 50
II.14. Piero's Treatment of the Third Dimension 50
II.15. The Column Problem 51
   Equidistant Line Segments 53
   Columns on Square Bases 54
   Cylindrical Columns 56
   Piero's Considerations 56
II.16. Piero's Plan and Elevation Construction 59
   The Origin of the Plan and Elevation Technique 59
   Piero's Construction 60
II.17. Piero's Cube 64
   Piero's Idea 66
   Piero's Illustrations 66
   Piero's Heads 71
II.18. Piero's Anamorphoses 71
II.19. Piero's Use of Perspective 75
II.20. Piero's Influence 79

Chapter III. Leonardo da Vinci 81

III.1. Leonardo and the History of Perspective 81
   Leonardo’s Trattato 82
   Leonardo’s Approach to Perspective 83
   Outline of This Chapter 84
III.2. Leonardo’s Various Concepts of Perspective 84
   Linear Perspective Versus Other Concepts of Perspective 85
   Natural Versus Accidental Perspective 86
   Composite and Simple Perspective 87
III.3. Visual Appearances and Perspective Representations 88
III.4. Leonardo on Visual Appearances of Lengths 89
   Leonardo’s Axiom and the Angle Axiom 89
   The Law of Inverse Proportionality 90
   Pacioli and the Law of Inverse Proportionality 94
   The Law of Inverse Proportionality and Euclid’s Theory 95
   Leonardo on the Appearance of a Rectangle 96
   The Appearance of the Vertical Boundaries 97
   The Appearance of Collinear Line Segments 98
III.5. Leonardo on Perspective Representations 100
   The Perspective Images of Particular Line Segments 101
   The Perspective Images of Collinear Line Segments 102
   Leonardo and the Column Problem 105
   Leonardo’s Appeal for a Large Viewing Distance 107
III.6. Leonardo and Curvilinear Perspective 107
III.7. Leonardo’s Doubts and Their Consequences 111
   Perspective and Visual Impressions 111
   Fixed Eye Point 111
   Leonardo’s Use of Perspective 112

Chapter IV. Italy in the Cinquecento 115

IV.1. The Italian Sixteenth-Century Perspectivists 115
IV.2. The Architectural, Painting, and Sculpting Traditions
   Gaurico 116
   Serlio 116
   Sirigatti, Cataneo, and Peruzzi 122
   Lomazzo 124

IV.3. A Mathematical Approach to Perspective –
   The Contributions by Vignola and Danti 125
   The First Edition of Vignola's Work on Perspective 125
   Vignola's Plan and Elevation Construction 126
   Vignola's Distance Point Construction 128
   Vignola's Comparison of His Two Methods 130
   Danti on Convergence Points 136

IV.4. Connection Between Perspective and Another Central
   Projection – Commandino's Contributions 138
   The Context of Commandino's Work 138
   Commandino's Constructions 141
   Commandino's Influence 145

IV.5. Another Mathematical Approach – Benedetti's Contributions 146
   Benedetti's Alberti Construction 146
   Benedetti on Pointwise Constructions 147
   Benedetti and Convergence Points 149
   Benedetti's Influence 152

IV.6. An Encyclopedia on Perspective – Barbaro's Book 152
   Barbaro's Sources 152
   Barbaro on the Regular Polyhedra 155

IV.7. The Italian Pre-1600 Contributions to Perspective 158

Chapter V. North of the Alps Before 1600 161

V.1. The Introduction of Perspective North of the Alps 161

V.2. Viator and His Followers 161
   Viator 162
   Ringelberg 166
   Cerceau 169

V.3. Cousin 172
   Cousin's Introduction of a Distance Point Construction 175
   Cousin's Use of Points of Convergence 178
   Cousin on the Column Problem 182

V.4. Dürer 183
   Dürer's Introduction to Perspective 183
   Dürer's Books 188
   Dürer's Plan and Elevation Construction 194
   Dürer's Enigmatic Method 197
   The Second Method as Described 199
   The Second Method as Illustrated 200
   The Second Method and Alberti Constructions 201
   The Second Method and a Distance Point Construction 202
   The Diagrams Illustrating the Second Method 204
   Finishing the Image by the Second Method 204
   Construction of the Side fg 205
Contents

A Perspective Instrument 285
An Arithmetical Example 285
VI.11. Stevin's Influence 287
The Knowledge of Stevin's Work Abroad 288
The Knowledge of Stevin's Work at Home 289
Conclusion 289

Chapter VII. The Dutch Development after Stevin 291

VII.1. A Survey of the Literature 291
VII.2. The Theory and Practice of Perspective 296
VII.3. The Work by Marolois 297
Marolois's Theory and Practice of Perspective 298
Marolois's Method of Construction 301
Marolois's Instrument 302
Shadows and Inverse Problems of Perspective 304
The Column Problem 308
Arithmetical Calculations 309
Marolois's Influence 309

VII.4. Van Hoogstraten's Perspective Box 309
The Left-Hand and Right-Hand Side Panels 313
The Bottom Panel 314
The Top Panel 316
The Back Panel 317

VII.5. Van Schooten's Revival of Stevin's Theory 317
Van Schooten's Intention and Inspiration 319
Georg Mohr 323
Abraham de Graaf 324
Hendrik van Houten 327

VII.6. The Problems of Reversing and Scaling 328
The Problem of Reversing 330
The Problem of Scaling 334
Reduced Distance 336

VII.7. 'sGravesande's Essay on Perspective 338
'sGravesande and His Work on Perspective 338
The Contents of 'sGravesande's Work 339
Camerae Obscurae 340
The Basic Theory 342
The Turned-In Eye Point 343
A Particular Line 345
'sGravesande's Basic Constructions 348
Oblique Picture Planes 351
'sGravesande's Examples 354
Shadows 357
Response to 'sGravesande's Work 359
The Audience for Books on Perspective 359

VII.8. Traces of Desargues's Method in Dutch Perspective 360
VII.9. Jelgerhuis and the Choice of Parameters 363
The Parameters of a Picture 364
Jelgerhuis's Choice 367

VII.10. The Dutch Scene 367
Chapter VIII. Italy after Guidobaldo

VIII.1. Waning Interest

VIII.2. Perspective in Textbooks on Architecture
   Seventeenth-Century Authors: Barca and Viola-Zanini
   Eighteenth-Century Authors: Amico, Vittone, Spampani, and Antonini
   The Galli-Bibienas and Piranesi

VIII.3. Perspective in Other Textbooks
   Textbooks on Stage Design – Chiaramonti and Sabbatini
   A Textbook on Useful Matters for Painters – Zaccolini
   A Textbook on the Theory of Vision – Diano
   A Textbook on Mixed Mathematics – Bettini
   A Textbook on Mathematics – Guarini

VIII.4. The Prospettiva Pratica Tradition
   Cigoli
   Contino
   Accolti
   Torricelli
   Troili
   Amato
   Quadri

VIII.5. Pozzo’s Influential Textbook
   Pozzo’s Methods
   Pozzo’s Virtual Dome
   A Vault As Picture Plane

VIII.6. A Special Approach to Perspective – Costa

VIII.7. Mathematical Approaches to Perspective
   Zanotti
   Stellini
   Torelli

VIII.8. The Later Italian Period

Chapter IX. France and the Southern Netherlands after 1600

IX.1. The Early Modern French Publications
   Perspective and Projective Geometry

IX.2. The Theory of Perspective Taught
   Aguilon and Mersenne
   Hérigone
   Bourdin, Dechales, and Tacquet
   Rohault and Ozanam
   Ozanam on Measure Points
   The Encyclopedias

IX.3. The Works of de Caus and Vaulezard
   De Caus
   Vaulezard on Cylindrical Mirror Anamorphoses
   Vaulezard on Perspective

IX.4. The Work of Aleaume and Migon
   The History of the Book by Aleaume and Migon
   Introduction of a Perspective Grid
Contents

Introduction of an Angle Scale 422
Methods Independent of Vanishing Points 424
Further Issues Treated by Aleaume and Migon 426
IX.5. Desargues’s Perspective Method 427
Desargues’s Avoidance of Vanishing Points 433
Theoretical Reflections in *La perspective* 436
Theoretical Reflections in *Aux théoriciens* 437
Conclusion on Desargues and Vanishing Points 441
Points at Infinity in Desargues’s Work on Perspective 442
IX.6. *Brouillon project* and Perspective 445
Cross Ratios 446
Projection of Conics 446
Two Traditions 447
IX.7. Perspectivists at War – and the Work of Dubreuil 448
Dubreuil 448
Desargues and Dubreuil 449
Dubreuil’s Comrades-in-Arms 451
IX.8. The Work of Niceron 452
Niceron’s Construction of an Anamorphic Grid 454
IX.9. Second Act of the Desargues Drama 457
Desargues’s Supporter Bosse 460
Bosse and the Royal Academy of Painting 460
IX.10. The 1660s and 1670s 465
Huret 465
Le Clerc 466
Bourgoing 467
IX.11. Perspective and the Educated Mathematician 470
IX.12. French Eighteenth-Century Literature on Perspective 471
Lamy 471
Bretez, Courtonne, Deidier, and Roy 474
Petitot and Curel 477
Lacaille 479
Jeaurat 482
Taylor’s Theory Introduced in France 482
Michel 484
Valenciennes 485
IX.13. The French Development 485

Chapter X. Britain 489

X.1. Starting Late 489
X.2. British Literature on Perspective Before Taylor 489
Wren, Moxon, and Salmon 490
Ditton 492
X.3. Taylor and His Work on Perspective 494
Taylor’s Background 494
Taylor’s Inspiration 496
Taylor’s Aim 496
Taylor’s Two Books on Perspective 498
## Contents

X.4. Taylor's Fundamental Concepts and Results
- Vanishing Points and Lines
- The Directing Plane

X.5. Taylor's Basic Constructions
- Pointwise Constructions
- Taylor's Inspiration from 'sGravesande

X.6. Taylor's Contributions to Plane Perspective Geometry
- Taylor's Solution to Problem 1
- Taylor's Solutions to Problems 2 and 3

X.7. Taylor's Contributions to Solid Perspective Geometry

X.8. Taylor's Examples of Drawing Figures in Perspective
- Constructions as an Intellectual Experiment
- A Direct Plan and Elevation Construction

X.9. Taylor's Treatment of Shadows

X.10. Taylor on Reflections

X.11. Taylor on Inverse Problems of Perspective
- Problems about Determining the Eye Point
- Problems Concerning the Shape of an Original Figure
- Determining the Eye Point as Well as the Shape

X.12. The Immediate Response to Taylor's Work

X.13. Taylor's Work in History

X.14. Hamilton's Comprehensive Work on Perspective
- Hamilton's Background
- Perspective and Conic Sections
- Hamilton on Linear Perspective
- Hamilton's Influence

X.15. Kirby and Highmore
- Kirby's Publications on Perspective
- Kirby's Main Work on Perspective
- Kirby's Inspiration
- Kirby on the Theory of Perspective
- Kirby on the Practice of Perspective
- Kirby and the Column Problem
- Kirby's Service to Taylor
- Highmore

X.16. The Taylor Tradition Continued
- Bardwell Protesting
- Fournier and Cowley Addressing Students at Military Academies
- Emerson, the Textbook Writer
- The Scientist Priestley Entering the Field
- Noble Attempting to Bridge the Gap Between Theory and Practice
- Malton and Son
- Clarke Presenting Perspective for Young Gentlemen
- Wood Writing for Painters
- Taylor's Influence on the Drawing of Chairs

X.17. Perspective in Textbooks on Mathematics
- Martin
- Muller
- Wright
Chapter XI. The German-Speaking Areas after 1600

XI.1. Categorization of the German Literature

XI.2. Perspective Instruments

Faulhaber and Bramer
Brunn and Scheiner
Halt
Hartnack
Meister and Hoffmann
Bischoff and Bürja

XI.3. Anamorphoses

Albrecht
Kircher and Schott
Leupold
Mathematischer Lust und Nutzgarten

XI.4. Perspective Presented for Practitioners

The Unknown Füllisch
The Philomath Haesell
The Painters Sandrart and Heinecke
The Architect and Drawer Schübeler
The Engraver Werner
The Master Carpenter Rödel
Gericke and Weidemann, Professors at the Academy of Art
The Theologian Horstig

XI.5. Mathematical Works on Perspective

The Wolffian Tradition
Weidler
Jena Scholars
Hennert and Lorenz
Segner and Bürja
Kästner's Analytical Approach
Kästner's General Theory
Karsten's Mastodon

XI.6. Traces of Lambert

XI.7. Perspective in the German Countries

Chapter XII. Lambert

XII.1. Lambert's Special Position

XII.2. Life and Work on Perspective

XII.3. Early Approach to Perspective
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>XII.4. The Contents of <em>Freye Perspektive</em></td>
<td>647</td>
</tr>
<tr>
<td>XII.5. Constructing Polygons in the Picture Plane</td>
<td>650</td>
</tr>
<tr>
<td>XII.6. Oblique Figures</td>
<td>655</td>
</tr>
<tr>
<td>XII.7. Shadows</td>
<td>661</td>
</tr>
<tr>
<td>XII.8. Reflections</td>
<td>665</td>
</tr>
<tr>
<td>XII.9. Parallel Projections</td>
<td>674</td>
</tr>
<tr>
<td>XII.10. Inverse Problems of Perspective</td>
<td>679</td>
</tr>
<tr>
<td>XII.11. Lambert's Practice of Perspective</td>
<td>682</td>
</tr>
<tr>
<td>XII.12. Ruler Geometry</td>
<td>689</td>
</tr>
<tr>
<td>Chapter XIII. Monge Closing a Circle</td>
<td>707</td>
</tr>
<tr>
<td>XIII.1. Monge and Descriptive Geometry</td>
<td>707</td>
</tr>
<tr>
<td>XIII.2. Monge and Linear Perspective</td>
<td>709</td>
</tr>
<tr>
<td>XIV.1. Opening Remarks</td>
<td>713</td>
</tr>
<tr>
<td>XIV.2. Local Approaches to Perspective</td>
<td>714</td>
</tr>
<tr>
<td>XIV.3. Perspective and Pure Mathematics</td>
<td>716</td>
</tr>
<tr>
<td>Chapter XIV. Summing Up</td>
<td>713</td>
</tr>
<tr>
<td>XIV.1. Opening Remarks</td>
<td>713</td>
</tr>
<tr>
<td>XIV.2. Local Approaches to Perspective</td>
<td>714</td>
</tr>
<tr>
<td>XIV.3. Perspective and Pure Mathematics</td>
<td>716</td>
</tr>
</tbody>
</table>

*Note: The page numbers for each section are provided as an example and may not correspond to the actual page numbers in the document.*
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interplay Between Perspective and Other Geometrical Disciplines</td>
<td>717</td>
</tr>
<tr>
<td>The Status of the Theory of Perspective</td>
<td>718</td>
</tr>
<tr>
<td>XIV.4. The Theory and Practice of Perspective</td>
<td>719</td>
</tr>
<tr>
<td>The Practitioners' Appreciation of the Theory of Perspective</td>
<td>719</td>
</tr>
<tr>
<td>Communication Between Theorists and Practitioners</td>
<td>719</td>
</tr>
<tr>
<td>The Usefulness of the Theory of Perspective</td>
<td>720</td>
</tr>
<tr>
<td>XIV.5. The Driving Forces Behind the Theory of Perspective</td>
<td>720</td>
</tr>
<tr>
<td>Appendix One. On Ancient Roots of Perspective</td>
<td>723</td>
</tr>
<tr>
<td>Optics</td>
<td>723</td>
</tr>
<tr>
<td>The Visual Pyramid and the Angle Axiom</td>
<td>723</td>
</tr>
<tr>
<td>The Remoteness Theorem</td>
<td>724</td>
</tr>
<tr>
<td>The Convergence Theorem</td>
<td>725</td>
</tr>
<tr>
<td>Optics and Perspective in Harmony</td>
<td>727</td>
</tr>
<tr>
<td>Cartography</td>
<td>727</td>
</tr>
<tr>
<td>Ptolemy's Geography</td>
<td>727</td>
</tr>
<tr>
<td>Ptolemy's <em>Planisphaerium</em></td>
<td>728</td>
</tr>
<tr>
<td>Scenography</td>
<td>728</td>
</tr>
<tr>
<td>Conclusion</td>
<td>730</td>
</tr>
<tr>
<td>Appendix Two. The Appearance of a Rectangle à la Leonardo da Vinci</td>
<td>731</td>
</tr>
<tr>
<td>The Curves for Three Different Distances</td>
<td>732</td>
</tr>
<tr>
<td>The Angle Between the Line Segments</td>
<td>734</td>
</tr>
<tr>
<td>Appendix Three. 'sGravesande Taking Recourse to the Infinitesimal Calculus to Draw a Column Base in Perspective</td>
<td>735</td>
</tr>
<tr>
<td>The First Step</td>
<td>736</td>
</tr>
<tr>
<td>The Infinitesimal and Limit Situation</td>
<td>737</td>
</tr>
<tr>
<td>The Perspective Image of the Visible Part of the Column Base</td>
<td>738</td>
</tr>
<tr>
<td>Appendix Four. The Perspective Sources Listed Countrywise in Chronological Order</td>
<td>739</td>
</tr>
<tr>
<td>Introduction</td>
<td>739</td>
</tr>
<tr>
<td>Italy</td>
<td>739</td>
</tr>
<tr>
<td>France and the Southern Netherlands</td>
<td>741</td>
</tr>
<tr>
<td>Germany, Austria, and Switzerland</td>
<td>742</td>
</tr>
<tr>
<td>The Northern Netherlands</td>
<td>744</td>
</tr>
<tr>
<td>Britain</td>
<td>745</td>
</tr>
<tr>
<td>First Bibliography. Pre-Nineteenth Century Publications on Perspective</td>
<td>747</td>
</tr>
<tr>
<td>Second Bibliography. Supplementary Literature</td>
<td>771</td>
</tr>
<tr>
<td>Index</td>
<td>795</td>
</tr>
<tr>
<td>Illustration Credits</td>
<td>811</td>
</tr>
</tbody>
</table>