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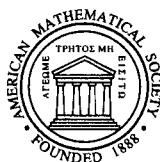
MATHEMATICAL MONOGRAPHS

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Hilbert C^* -Modules

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