Period mappings and differential equations. From $\mathbb{C}$ to $\mathbb{C}_p$

Tôhoku-Hokkaidô lectures in Arithmetic Geometry
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