## 4. Analytical Methods for Simply Connected Bounded Domains

4.2. Explicit Formulas for Eigenfrequencies of Round Disc ........................................ 107
4.3. Some Variational Principles for Eigenvalues .................................................. 110
4.4. Weyl–Carleman Theory of Asymptotic Distribution of Large Eigenvalues ...................... 115
4.5. Exact Explicit Results for Some Polygons .................................................. 119
4.6. Explicit Analytical Results for Some Polyhedra ........................................ 125

## 5. Integral Equations in Diffraction by Linear Obstacles

5.1. Integral Operators in Diffraction by Linear Screen and by a Gap in the Screen .......... 133
5.2. Operator Equation in Diffraction Problem on a Crack in Unbounded Elastic Medium .......................................................... 138
5.3. High-Frequency Asymptotics in Diffraction by Linear Obstacles in Unbounded Medium .......................................................... 142
5.4. High-Frequency Asymptotics for Diffraction by Linear Obstacles in Open Waveguides .......................................................... 145
5.5. High-Frequency Diffraction by a Linear Discontinuity in the Waveguide .................... 151
5.6. Waves in Elastic Half-Space. Factorization of the Rayleigh Function ....................... 157
5.7. Integral Equation of the Mixed Boundary Value Problem for Elastic Layer ................ 160

## 6. Short-Wave Asymptotic Methods on the Basis of Multiple Integrals

6.1. Schoch's Method: Exact Representation of 3D Wave Fields by One-Dimensional Quadratures .......................................................... 165
6.2. High-Frequency Wave Fields in Elastic Half-Space ........................................ 169
6.3. Asymptotic Nature of the Geometrical Diffraction Theory ........................................ 171
6.4. High-Frequency Diffraction with Re-Reflections ........................................ 175
6.5. Application: Examples of High-Frequency Multiple Diffraction ................................ 180
6.6. Application: Physical Diffraction Theory for Nonconvex Obstacles .............................. 184
6.7. Short-Wave Integral Operator in Diffraction by a Flaw in Elastic Medium .................... 186
6.8. High-Frequency Asymptotics of Integral Operator in a Three-Dimensional Diffraction Theory .......................................................... 190

## 7. Inverse Problems of the Short-Wave Diffraction

7.1. Some Basic Results in a Local Differential Geometry of Smooth Convex Surfaces .......................................................... 195
7.2. Reducing Inverse Problem of the Short-Wave Diffraction to Minkowski Problem .... 199
7.3. Explicit Results for a Differential Operator of the 2D Inverse Problem ...................... 201
7.4. Exact Explicit Inversion of the Basic Operator in the Case of Axial Symmetry .............. 203
7.5. Nonlinear Differential Operator of the Three-Dimensional Inverse Problem .................. 205
7.6. Reconstruction of Nonconvex Obstacles in the High-Frequency Range: 2D Case ........... 208
7.7. Reconstruction of Nonconvex Obstacles in the High-Frequency Range: 3D Case ........... 213
## Contents

8. **Ill-Posed Equations of Inverse Diffraction Problems for Arbitrary Boundary**  219  
  8.1. Ill-Posed Problems for Operator Equations of the First Kind: General Properties 219  
  8.2. Regularization of Ill-Posed Problems with the Help of Smoothing Functional 222  
  8.3. Iterative Methods for Operator Equations of the First Kind 226  
  8.4. Comparison of Various Methods for Reconstruction of the Scatterer Geometry 231  
  8.5. General Inverse Diffraction Problem: Combination of Iterations and Smoothing 235  
  8.6. A Correct Treatment of Ill-Posed Boundary Equations in Acoustics of Closed Regions 242  
  8.7. Ill-Posed Method of Auxiliary Sources in Diffraction Theory 248  
  8.8. A Method of Global Random Search in Inverse Problems 251  
  8.9. Ill-Posed Problem on Reconstruction of Convex Hull of the Obstacle in Acoustic Medium 253  

9. **Numerical Methods for Irregular Operator Equations** 259  
  9.1. Steepest Descent Method: Stability and Improvement of the Convergence 259  
  9.2. Galerkin Methods for Integral Equations of the First Kind with Weakly Singular Kernels 263  
  9.3. Integral Equations of the Physical Diffraction Theory in the Case of Nonconvex Obstacles 268  
  9.4. Numerical Methods in Singular Integral Equations with the Cauchy-Type Kernel 272  
  9.5. Numerical Methods for Hyper-Singular Integral Equations 276  

References 281  

Index 287