Oscillating Patterns in Image Processing and Nonlinear Evolution Equations

The Fifteenth
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Memorial Lectures

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