

# Algebraic Statistics

Computational Commutative  
Algebra in Statistics

GIOVANNI PISTONE  
EVA RICCOMAGNO  
HENRY P. WYNN

CHAPMAN & HALL/CRC

Boca Raton London New York Washington, D.C.

---

# Contents

---

<b>List of figures</b>	<b>ix</b>
<b>List of tables</b>	<b>xi</b>
<b>Preface</b>	<b>xiii</b>
<b>Notation</b>	<b>xv</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Outline	1
1.2 Computer Algebra	5
1.3 An example: the $2^{3-1}$ fractional factorial design	10
<b>2 Algebraic models</b>	<b>15</b>
2.1 Models	16
2.2 Polynomial ideals	17
2.3 Term-orderings	19
2.4 Division algorithm	22
2.5 Hilbert basis theorem	23
2.6 Varieties and equations	25
2.7 Gröbner bases	27
2.8 Properties of a Gröbner basis	29
2.9 Elimination theory	31
2.10 Polynomial functions and quotients by ideals	33
2.11 Hilbert function	35
2.12 Further topics	36
<b>3 Gröbner bases in experimental design</b>	<b>43</b>
3.1 Designs and design ideals	43
3.2 Computing the Gröbner basis of a design	44
3.3 Operations with designs	47
3.4 Examples	48
3.5 Span of a design	50
3.6 Models and identifiability: quotients	53

3.7	Confounding of models	54
3.8	Further examples	56
3.9	The fan of an experimental design	60
3.10	Minimal and maximal fan designs	63
3.11	Hilbert functions and fans for graded ordering	65
3.12	Subsets and algorithms	66
3.13	Regression analysis	71
3.14	Non-polynomial models	72
<b>4</b>	<b>Two-level factors: logic, reliability, design</b>	<b>75</b>
4.1	The binary case: Boolean representations	75
4.2	Gröbner bases and Boolean ideals	78
4.3	Logic and learning	80
4.4	Reliability: coherent systems as minimal fan designs	81
4.5	Inclusion-exclusion and tube theory	83
4.6	Two-level factorial design: contrasts and orthogonality	90
<b>5</b>	<b>Probability</b>	<b>95</b>
5.1	Random variables on a finite support	96
5.2	The ring of random variables	97
5.3	Matrix representation of $\mathcal{L}(D, \mathcal{K})$	99
5.4	Uniform probability	101
5.5	Probability densities	103
5.6	Image probability and marginalisation	106
5.7	Conditional expectation	108
5.8	Algebraic representation of exponentials	111
5.9	Exponential form of a probability	113
<b>6</b>	<b>Statistical modeling</b>	<b>119</b>
6.1	Introduction	119
6.2	Statistical models	120
6.3	Generating functions and exponential submodels	128
6.4	Likelihoods and sufficient statistics	131
6.5	Score function and information	135
6.6	Estimation: lattice case	136
6.7	Finitely generated cumulants	138
6.8	Estimating functions	139
6.9	An extended example	140
6.10	Orthogonality and toric ideals	149
	<b>References</b>	<b>155</b>
	<b>Index</b>	<b>159</b>