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Acoustic and Electromagnetic Equations

Integral Representations
for Harmonic Problems



Springer

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