Henk J.M. Bos

Redefining Geometrical Exactness

Descartes' Transformation of the Early Modern Concept of Construction

With 95 Illustrations

Springer
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