

William Fulton

Intersection Theory

Second Edition



Springer

Contents

Introduction	1
Chapter 1. Rational Equivalence	6
Summary	6
1.1 Notation and Conventions	6
1.2 Orders of Zeros and Poles	8
1.3 Cycles and Rational Equivalence	10
1.4 Push-forward of Cycles	11
1.5 Cycles of Subschemes	15
1.6 Alternate Definition of Rational Equivalence	15
1.7 Flat Pull-back of Cycles	18
1.8 An Exact Sequence	21
1.9 Affine Bundles	22
1.10 Exterior Products	24
Notes and References	25
Chapter 2. Divisors	28
Summary	28
2.1 Cartier Divisors and Weil Divisors	29
2.2 Line Bundles and Pseudo-divisors	31
2.3 Intersecting with Divisors	33
2.4 Commutativity of Intersection Classes	35
2.5 Chern Class of a Line Bundle	41
2.6 Gysin Map for Divisors	43
Notes and References	45
Chapter 3. Vector Bundles and Chern Classes	47
Summary	47
3.1 Segre Classes of Vector Bundles	47
3.2 Chern Classes	50
3.3 Rational Equivalence on Bundles	64
Notes and References	68
Chapter 4. Cones and Segre Classes	70
Summary	70
4.1 Segre Class of a Cone	70

4.2 Segre Class of a Subscheme	73
4.3 Multiplicity Along a Subvariety	79
4.4 Linear Systems	82
Notes and References	85
Chapter 5. Deformation to the Normal Cone	86
Summary	86
5.1 The Deformation	86
5.2 Specialization to the Normal Cone	89
Notes and References	90
Chapter 6. Intersection Products	92
Summary	92
6.1 The Basic Construction	93
6.2 Refined Gysin Homomorphisms	97
6.3 Excess Intersection Formula	102
6.4 Commutativity	106
6.5 Functoriality	108
6.6 Local Complete Intersection Morphisms	112
6.7 Monoidal Transforms	114
Notes and References	117
Chapter 7. Intersection Multiplicities	119
Summary	119
7.1 Proper Intersections	119
7.2 Criterion for Multiplicity One	126
Notes and References	127
Chapter 8. Intersections on Non-singular Varieties	130
Summary	130
8.1 Refined Intersections	130
8.2 Intersection Multiplicities	137
8.3 Intersection Ring	140
8.4 Bézout's Theorem (Classical Version)	144
Notes and References	151
Chapter 9. Excess and Residual Intersections	153
Summary	153
9.1 Equivalence of a Connected Component	153
9.2 Residual Intersection Theorem	160
9.3 Double Point Formula	165
Notes and References	171

Chapter 10. Families of Algebraic Cycles	175
Summary	175
10.1 Families of Cycle Classes	176
10.2 Conservation of Number	180
10.3 Algebraic Equivalence	185
10.4 An Enumerative Problem	187
Notes and References	193
Chapter 11. Dynamic Intersections	195
Summary	195
11.1 Limits of Intersection Classes	196
11.2 Infinitesimal Intersection Classes	198
11.3 Limits and Distinguished Varieties	200
11.4 Moving Lemmas	205
Notes and References	209
Chapter 12. Positivity	210
Summary	210
12.1 Positive Vector Bundles	211
12.2 Positive Intersections	218
12.3 Refined Bézout Theorem	223
12.4 Intersection Multiplicities	227
Notes and References	234
Chapter 13. Rationality	235
Summary	235
Notes and References	241
Chapter 14. Degeneracy Loci and Grassmannians	242
Summary	242
14.1 Localized Top Chern Class	244
14.2 Gysin Formulas	247
14.3 Determinantal Formula	249
14.4 Thom-Porteous Formula	254
14.5 Schur Polynomials	263
14.6 Grassmann Bundles	266
14.7 Schubert Calculus	271
Notes and References	278
Chapter 15. Riemann-Roch for Non-singular Varieties	280
Summary	280
15.1 Preliminaries.	280
15.2 Grothendieck-Riemann-Roch Theorem	286
15.3 Riemann-Roch Without Denominators	296

15.4 Blowing up Chern Classes	298
Notes and References	302
Chapter 16. Correspondences	305
Summary	305
16.1 Algebra of Correspondences	305
16.2 Irregular Fixed Points	315
Notes and References	318
Chapter 17. Bivariant Intersection Theory	319
Summary	319
17.1 Bivariant Rational Equivalence Classes	320
17.2 Operations and Properties	322
17.3 Homology and Cohomology	324
17.4 Orientations	326
17.5 Monoidal Transforms	332
17.6 Residual Intersection Theorem	333
Notes and References	337
Chapter 18. Riemann-Roch for Singular Varieties	339
Summary	339
18.1 Graph Construction	340
18.2 Riemann-Roch for Quasi-projective Schemes	348
18.3 Riemann-Roch for Algebraic Schemes	353
Notes and References	368
Chapter 19. Algebraic, Homological and Numerical Equivalence	370
Summary	370
19.1 Cycle Map	371
19.2 Algebraic and Topological Intersections	378
19.3 Equivalence on Non-singular Varieties	385
Notes and References	391
Chapter 20. Generalizations	393
Summary	393
20.1 Schemes Over a Regular Base Scheme	393
20.2 Schemes Over a Dedekind Domain	397
20.3 Specialization	398
20.4 Tor and Intersection Products	401
20.5 Higher K -theory	403
Notes and References	404
Appendix A. Algebra	406
Summary	406
A.1 Length.	406

A.2 Herbrand Quotients	407
A.3 Order Functions	411
A.4 Flatness	413
A.5 Koszul Complexes	414
A.6 Regular Sequences	416
A.7 Depth	418
A.8 Normal Domains	419
A.9 Determinantal Identities	419
Notes and References	425
 Appendix B. Algebraic Geometry (Glossary)	 426
B.1 Algebraic Schemes	426
B.2 Morphisms	427
B.3 Vector Bundles	430
B.4 Cartier Divisors	431
B.5 Projective Cones and Bundles	432
B.6 Normal Cones and Blowing Up	435
B.7 Regular Imbeddings and l.c.i. Morphisms	437
B.8 Bundles on Imbeddable Schemes	439
B.9 General Position	440
 Bibliography	 442
 Notation	 462
 Index	 464