Hidden Markov and Other Models for Discrete-valued Time Series

Iain L. MacDonald

University of Cape Town
South Africa

and

Walter Zucchini

University of Göttingen
Germany
## Contents

### Preface  xiii

### PART ONE  Survey of models  1

1 **A survey of models for discrete-valued time series**  3  
   1.1 Introduction: the need for discrete-valued time  
   series models  3  
   1.2 Markov chains  6  
   1.2.1 Saturated Markov chains  6  
   1.2.2 A nonhomogeneous Markov chain model for  
   binary time series  11  
   1.3 Higher-order Markov chains  12  
   1.4 The DARMA models of Jacobs and Lewis  17  
   1.5 Models based on thinning  21  
   1.5.1 Models with geometric marginal  21  
   1.5.2 Models with negative binomial marginal  23  
   1.5.3 Models with Poisson marginal  25  
   1.5.4 Models with binomial marginal  29  
   1.5.5 Results not based on any explicit distribu-  
   tional assumption  30  
   1.6 The bivariate geometric models of Block, Langberg  
   and Stoffer  32  
   1.6.1 Moving average models with bivariate geo-  
   metric distribution  32  
   1.6.2 Autoregressive and autoregressive moving  
   average models with bivariate geometric  
   distribution  34  
   1.7 Markov regression models  37  
   1.8 Parameter-driven models  42  
   1.9 State-space models  45
PART TWO  Hidden Markov models

2  The basic models
   2.1  Introduction
   2.2  Some theoretical aspects of hidden Markov models in speech processing
   2.3  Hidden Markov time series models: definition and notation
   2.4  Correlation properties
       2.4.1  The autocorrelation function of a Poisson-hidden Markov model
       2.4.2  The autocorrelation function of a binomial-hidden Markov model
       2.4.3  The partial autocorrelation function
   2.5  Evaluation of the likelihood function
   2.6  Distributional properties
       2.6.1  Marginal, joint and conditional distributions of the observations
       2.6.2  The Markov chain conditioned on the observations
       2.6.3  Runlength distributions for binary hidden Markov models
   2.7  Parameter estimation
       2.7.1  Computing maximum likelihood estimates
       2.7.2  Asymptotic properties of maximum likelihood estimators
       2.7.3  Use of the parametric bootstrap
   2.8  Identification of outliers
   2.9  Reversibility
   2.10 Discussion

3  Extensions and modifications
   3.1  Introduction
   3.2  Models based on a second-order Markov chain
   3.3  Multinominal-hidden Markov models
       3.3.1  The likelihood
       3.3.2  Marginal properties and cross-correlations
       3.3.3  A model for categorical time series
   3.4  Multivariate models
3.4.1 The likelihood function for multivariate models 122
3.4.2 Cross-correlations of models assuming contemporaneous conditional independence 122
3.4.3 Cross-correlations of models not assuming contemporaneous conditional independence 124
3.4.4 Multivariate models with time lags 125
3.4.5 Multivariate models in which some variables are discrete and others continuous 126
3.5 Models with state-dependent probabilities depending on covariates 128
3.6 Models in which the Markov chain is homogeneous but not assumed stationary 129
3.7 Models in which the Markov chain is nonhomogeneous 130
3.8 Joint models for the numbers of trials and the numbers of successes in those trials 133
3.9 Discussion 135

4 Applications 137
4.1 Introduction 137
4.2 The durations of successive eruptions of the Old Faithful geyser 138
  4.2.1 Markov chain models 138
  4.2.2 Hidden Markov models 140
  4.2.3 Comparison of models 144
  4.2.4 Forecast distributions 146
4.3 Epileptic seizure counts 146
4.4 Births at Edendale hospital 152
  4.4.1 Models for the proportion Caesarean 152
  4.4.2 Models for the total number of deliveries 159
  4.4.3 Conclusion 161
4.5 Locomotory behaviour of *Locusta migratoria* 162
  4.5.1 Multivariate models 163
  4.5.2 Univariate models 166
  4.5.3 Conclusion 167
4.6 Wind direction at Koeberg 168
  4.6.1 Three hidden Markov models for hourly averages of wind direction 168
  4.6.2 Model comparisons and other possible models 173
  4.6.3 Conclusion 176
4.7 Evapotranspiration 177
4.8 Thinly traded shares on the Johannesburg Stock Exchange

4.8.1 Univariate models 179
4.8.2 Multivariate models 180
4.8.3 Discussion 182

4.9 Daily rainfall at Durban 184

4.10 Homicides and suicides, Cape Town, 1986–1991 191

4.10.1 Models for firearm homicides as a proportion of all homicides, suicides and legal intervention homicides 191
4.10.2 Models for the number of firearm homicides 194
4.10.3 Firearm homicides as a proportion of all homicides, and firearm suicides as a proportion of all suicides 195
4.10.4 Models for the proportions in each of the five categories of death 200

4.11 Conclusion 201

Appendices 203

A Proofs of results used in the derivation of the Baum–Welch algorithm 203
B Data 207

References 217

Author index 227

Subject index 231