Mathematics of Climate Modeling

Valentin P. Dymnikov
Aleksander N. Filatov

Birkhäuser
Boston • Basel • Berlin
CONTENTS

PREFACE .................................................................................. vii
INTRODUCTION .......................................................................... ix

Chapter 1. DYNAMICAL SYSTEMS. ATTRACTORS,
INERTIAI MEASURES ................................................................. 1
1.1 Metric Spaces. Compactness ................................................. 5
1.2 Dynamical Systems. Main Properties ................................. 9
1.3 Invariant Sets ..................................................................... 14
1.4 Classification of Motions ...................................................... 16
1.5 Recurrence of Domains ....................................................... 23
1.6 Measure. Krylov-Bogolyubov Theorem .............................. 29
1.7 Dynamical Systems with Invariant Measure ..................... 41
1.8 Nonlinear Dissipative Systems .......................................... 52
1.9 Inertial Manifolds of Dissipative Systems ......................... 67

Chapter 2. NON-AUTONOMOUS DISSIPATIVE SYSTEMS,
THEIR ATTRACTOR AND AVERAGING ............................... 77
2.1 Introduction ........................................................................ 77
2.2 Processes and their Attractors. Kernel of Processes, ......... 81
Section of Kernel ..................................................................... 81
2.3 Families of Processes and their Attractors ...................... 85
2.4 Family of Processes and Semigroups ................................. 88
2.5 Averaging of Nonlinear Dissipative Systems. Closeness
between Attractors of Original and Averaged Systems ........ 93
2.6 On Closeness of Solutions of Original and Averaged
Nonlinear Dissipative Systems on Infinite Time Interval ....... 102

Chapter 3. ANALYSIS OF BAROTROPIC MODEL ................. 109
3.1 Existence of Global Attractor ............................................. 109
3.2 Estimate of Dimension of Attractor ................................. 105
3.3 Statistical Solutions and Invariant Measures on Attractor 107
3.4 Estimate of Attractor Dimension with Respect to
Orography ............................................................................. 121
3.5 Galerkin Approximations .................................................. 125
3.6 Existence of Inertial Manifold ............................................ 126

Chapter 4. DISCRETIZATION OF SYSTEMS POSSESSING
ATTRACTOR ................................................................. 131
4.1 Discretization of Systems Possessing Inertial Manifolds ... 132
4.2 Time-Space Discretization of Systems Possessing Attractor 133
4.3 Globally Stable Difference Schemes for Barotropic
Vorticity Equation ............................................................... 143
Chapter 5. NUMERICAL STUDY OF STRUCTURE OF ATTRACTOR GENERATED BY BAROTROPIC EQUATIONS ON SPHERE ........................................... 171
  5.1 Equations and Parameters of Model. Methods of Solving of Stationary and Nonstationary Problems ............................. 176
  5.2 Statistical Stationary Solution and Stationary Points .......... 180
  5.3 Lyapunov Exponents and Attractor Dimension ................. 287
  5.4 Analysis of Analytical Estimates of Attractor Dimension of Barotropic Atmospheric Equations ............................... 188
Chapter 6. TWO-LAYER BAROCLINIC MODEL .................. 189
  6.1 Two-Layer Baroclinic Model .................................. 193
  6.2 Estimate of Attractor Dimension ............................... 203
Chapter 7. INVESTIGATION OF STRUCTURE OF CLIMATE ATTRACTORS BY OBSERVED DATA SERIES ........ 211
  7.1. Correlation Dimension of Attractor .......................... 213
  7.2. Calculation of Lyapunov Exponents ......................... 216
  7.3 Statistically Independent Degrees of Freedom and Attractor Dimension .............................................................. 217
Chapter 8. REGIMES OF ATMOSPHERE CIRCULATION ... 221
  8.1 Definition of Atmosphere Circulation Regimes ............... 221
  8.2 Dynamical Theory of Two-Regime Barotropic Circulation 223
  8.3. Statistical Theory of Two-Regime Barotropic Circulation 229
  8.4 S-Regimes of Atmosphere Circulation .......................... 233
Chapter 9. SOLVABILITY OF OCEAN AND ATMOSPHERE MODELS ................................................................. 235
  9.1 Introduction ..................................................... 235
  9.2 Solvability of Ocean and Atmosphere Models in Bounded Domains ................................................................. 236
  9.3 Solvability of Ocean and Atmosphere Models on Sphere in p-System of Coordinates ............................................. 246
BIBLIOGRAPHY .......................................................... 247
INDEX ........................................................................ 259