

# **Fuzzy Rule-Based Modeling with Applications to Geophysical, Biological and Engineering Systems**

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