The Finite Element Analysis of Shells – Fundamentals

Second Edition
1. Introduction .................................................. 1
  1.1 Shells: from Nature to Engineering Designs ................. 1
  1.2 The Finite Element Analysis of Shells as Approached in this
      Book ....................................................... 4

2. Geometrical Preliminaries ..................................... 9
   2.1 Vectors and Tensors in Three-Dimensional Curvilinear Coor-
       dinates .................................................. 9
      2.1.1 Vectors and tensors .................................. 9
      2.1.2 Covariant and contravariant bases. Metric tensor ....... 11
      2.1.3 Curvilinear coordinate systems ....................... 17
      2.1.4 Covariant differentiation ........................... 20
   2.2 The Shell Geometry ....................................... 23
      2.2.1 Geometric definition of a shell ....................... 23
      2.2.2 Differential geometry on the midsurface ............... 25
      2.2.3 3D differential geometry for shells ................... 37

3. Elements of Functional and Numerical Analysis ................ 41
   3.1 Sobolev Spaces and Associated Norms ....................... 41
      3.1.1 General concepts in vector spaces .................... 42
      3.1.2 $L^2$ and other Sobolev spaces ....................... 48
   3.2 Variational Formulations and Finite Element Approximations 58
      3.2.1 Basic error estimates for displacement-based and mixed
            formulations ........................................... 59
      3.2.2 Interpolation and $a$ priori error estimates .......... 88
      3.2.3 Effect of numerical integration ....................... 93

4. Shell Mathematical Models ..................................... 95
   4.1 Shell Kinematics .......................................... 95
   4.2 Derivation of Shell Models ............................... 99
      4.2.1 The "basic shell model" ................................ 100
      4.2.2 The "shear-membrane-bending model" .................. 103
      4.2.3 The "membrane-bending model" ......................... 104
      4.2.4 Plate models ......................................... 107
4.2.5 Higher-order shell models, and the 3D-shell model... 110
4.3 Mathematical Analysis of the Shell Models .................. 114
  4.3.1 Analysis of the s-m-b shell model .................. 114
  4.3.2 Analysis of the m-b shell model .................. 123
  4.3.3 Analysis of the basic shell model .................. 125
  4.3.4 Analysis of the 3D-shell model .................. 130

5. Asymptotic Behaviors of Shell Models .................. 135
  5.1 General Asymptotic Analysis .................. 136
    5.1.1 Non-inhibited pure bending .................. 143
    5.1.2 Inhibited pure bending .................. 146
    5.1.3 Summary of asymptotic behaviors .................. 150
    5.1.4 Comparison of asymptotic behaviors for specific shell models .................. 152
  5.2 Analysis of the Subspace of Pure Bending Displacements .................. 156
    5.2.1 Elliptic surfaces .................. 157
    5.2.2 Hyperbolic surfaces .................. 158
    5.2.3 Parabolic surfaces .................. 159
  5.3 Influence of the Loading .................. 161
    5.3.1 Effect of the loadings that do not activate the pure bending displacements .................. 161
    5.3.2 Effect of non-admissible membrane loadings .................. 166
  5.4 Asymptotic Analysis of the 3D-Based Shell Models .................. 179
    5.4.1 Asymptotic analysis of the basic shell model .................. 180
    5.4.2 Asymptotic analysis of the 3D-shell model .................. 192
  5.5 Asymptotic Considerations in Dynamic Analysis .................. 208
    5.5.1 Non-inhibited pure bending .................. 208
    5.5.2 Inhibited pure bending .................. 209
    5.5.3 Detailed numerical illustration for a clamped cylinder .................. 210

6. Displacement-Based Shell Finite Elements .................. 219
  6.1 Discretizations of Shell Mathematical Models .................. 219
  6.2 Facet-Shell Elements .................. 224
  6.3 General Shell Elements .................. 228
  6.4 3D-Shell Elements .................. 253

7. Influence of the Thickness in the Finite Element Approximation .................. 259
  7.1 Numerical Locking in Thin Structures .................. 260
  7.2 Treatments of Numerical Locking by Mixed Formulations .................. 266
    7.2.1 Basic principles: the Timoshenko beam example .................. 267
    7.2.2 Applications to the Reissner-Mindlin plate model .................. 276
    7.2.3 Basic principles of stabilized mixed formulations .................. 291
    7.2.4 MITC plate elements .................. 295
  7.3 Specific Difficulties Arising in the Analysis of Shells .................. 304
8. Towards the Formulation of Effective General Shell Elements .................................................. 315
  8.1 Guidelines for Assessing and Improving the Reliability of Shell Finite Elements ......................... 315
    8.1.1 Considerations on proper selection and use of test problems ........................................ 315
    8.1.2 Proposed set of test problems ........................................ 323
  8.2 Formulation of MITC Shell Elements .......................... 326
    8.2.1 Formulation of quadrilateral MITC elements .......... 326
    8.2.2 Formulation of triangular MITC elements .......... 327
    8.2.3 Insight into MITC shell formulations ................ 332
    8.2.4 Considerations regarding 3D-shell elements .......... 336
  8.3 Assessment Results ........................................ 338
    8.3.1 Shell elements used in plate bending .......... 338
    8.3.2 Axisymmetric hyperboloid .............................. 344

9. On the Nonlinear Analysis of Shells .......................... 365
  9.1 The Incremental Analysis to Obtain Nonlinear Response Solutions ........................................ 365
  9.2 The Finite Element Discretization of a Shell for General Nonlinear Analysis ................................. 367
  9.3 The Fundamental Considerations of Linear Analysis Used in Nonlinear Analysis .............................. 370
  9.4 Demonstrative Solutions ........................................ 372
    9.4.1 The “Myth of No-Locking” in nonlinear analysis of shells ........................................ 372
    9.4.2 Large deformation analysis of a simply-supported plate 375
    9.4.3 Nonlinear analysis of thick cantilever beam .......... 376
    9.4.4 Contact analysis of Scordelis-Lo roof .............. 377
    9.4.5 Crash analysis of a tube ............................. 380

A. Tables of symbols ........................................ 383
  A.1 Latin Symbols ........................................ 383
  A.2 Greek Symbols ........................................ 386
  A.3 Special Symbols ........................................ 387

B. Some Useful Mathematical Formulas .......................... 389

C. Distributions: Basic Definitions and Properties .......... 391

Bibliography ........................................ 395

Index ........................................ 407