PLASTICITY AND GEOMECHANICS

R. O. DAVIS
University of Canterbury

A. P. S. SELVADURAI
McGill University
# Contents

**Preface**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress and strain</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Soil mechanics and continuum mechanics</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Sign conventions</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Deformation and strain</td>
<td>5</td>
</tr>
<tr>
<td>1.5 Strain compatibility</td>
<td>10</td>
</tr>
<tr>
<td>1.6 Forces and tractions</td>
<td>11</td>
</tr>
<tr>
<td>1.7 The stress matrix</td>
<td>12</td>
</tr>
<tr>
<td>1.8 Principal stresses</td>
<td>14</td>
</tr>
<tr>
<td>1.9 Mohr circles</td>
<td>16</td>
</tr>
<tr>
<td>1.10 The effective stress principle</td>
<td>20</td>
</tr>
<tr>
<td>1.11 Equilibrium</td>
<td>22</td>
</tr>
<tr>
<td>Further reading</td>
<td>25</td>
</tr>
<tr>
<td>Exercises</td>
<td>25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic and inelastic material behaviour</td>
<td>27</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>27</td>
</tr>
<tr>
<td>2.2 Hooke's law</td>
<td>28</td>
</tr>
<tr>
<td>2.3 Values for elastic constants</td>
<td>31</td>
</tr>
<tr>
<td>2.4 Solution of problems in elasticity</td>
<td>33</td>
</tr>
<tr>
<td>2.5 Plane elasticity</td>
<td>34</td>
</tr>
<tr>
<td>2.6 Indications of inelastic behaviour</td>
<td>38</td>
</tr>
<tr>
<td>2.7 The oedometer test</td>
<td>44</td>
</tr>
<tr>
<td>2.8 The triaxial test</td>
<td>46</td>
</tr>
<tr>
<td>Further reading</td>
<td>49</td>
</tr>
<tr>
<td>Exercises</td>
<td>50</td>
</tr>
</tbody>
</table>
3 Yield
  3.1 Introduction 52
  3.2 Principal stress space 53
  3.3 Yield surfaces for metals 58
  3.4 The Coulomb yield criterion 60
  3.5 Modifications to Coulomb's criterion 65
  3.6 The Cambridge models 68
  3.7 Two-dimensional yield loci 72
  3.8 Example – plane strain 75
     Further reading 79
     Exercises 80

4 Plastic flow
  4.1 Introduction 83
  4.2 Normality 84
  4.3 Associated flow rules 90
  4.4 Example – plane strain 92
  4.5 Non-associated flow 94
  4.6 A loading criterion 95
  4.7 A complete stress–strain relationship 97
  4.8 The pressuremeter problem 99
     Further reading 106
     Exercises 107

5 Collapse load theorems
  5.1 Introduction 109
  5.2 The theorems 110
  5.3 Discontinuities of stress and deformation 112
  5.4 A vertical cut 116
  5.5 Shallow foundation – lower bound 120
  5.6 Shallow foundation – upper bound 127
  5.7 Shallow foundation – discussion 133
  5.8 Retaining walls 137
  5.9 Arching 141
  5.10 Non-associated flow and the upper bound theorem 144
     Further reading 148
     Exercises 149

6 Slip line analysis
  6.1 Introduction 152
  6.2 Two-dimensional stress states 153
## Contents

6.3  Slip lines 155  
6.4  Slip line geometries 157  
6.5  Some simple problems 160  
6.6  Frictional materials 169  
6.7  Effects of gravity 175  
6.8  The velocity field 177  
Further reading 182  
Exercises 183  

7  Work hardening and modern theories for soil behaviour 185  
7.1  Introduction 185  
7.2  Work hardening for metals 187  
7.3  Cam Clay 190  
7.4  Beyond Cam Clay 201  
7.5  Last words 211  
Further reading 212  
Exercises 213  

Appendices 215  
A  Non-Cartesian coordinate systems 215  
B  Mohr circles 228  
C  Principles of virtual work 241  
D  Extremum principles 246  
E  Drucker’s stability postulate 255  
F  The associated flow rule 259  
G  A uniqueness theorem for elastic–plastic deformation 263  
H  Theorems of limit analysis 269  
I  Limit analysis and limiting equilibrium 277  

Index 279